Unit 3 Boolean Algebra (Continued)

3.1 Multiplying Out and Factoring Expressions

$X (Y + Z) = XY + XZ$
$(X + Y)(X + Z) = X + YZ$ last Unit

$(X + Y)(X' + Z) = XZ + X'Y$

Proof: *If* $X = 0$
$Y(1 + Z) = Y$
$Y = Y$

If $X = 1$
$(1 + Y)Z = Z$
$Z = Z$

Uses:
Factoring

$AB + A'C = (A + C) (A' + B)$

Multiplying Out

$(Q + AB') (C'D + Q') = QC'D + Q'AB'$

Note: *Generally the above equation should be used when multiplying out prior to multiply through.*

Ex. $(A+B+C') (A+B+D) (A+B+E) (A+D'+E)(A'+C)$

Factoring

AC + A’BD’ + A’BE + A’C’DE

3.2 Exclusive OR and Equivalence Operations

Exclusive OR ($\oplus$)

$0 \oplus 0 = 0$ $\quad$ $1 \oplus 0 = 1$

$0 \oplus 1 = 1$ $\quad$ $1 \oplus 1 = 0$

Truth table

X	Y	X ⊕ Y
0	*0*	*0*
0	*1*	*1*
1	*0*	*1*
1	*1*	*0*

X ⊕ Y = 1 iff X = 1 or Y = 1, but not both

Rewritten X ⊕ Y = 1 if X = 0 and Y = 1 OR X = 1 and Y = 0

$$X \oplus Y = X'Y + XY'$$

Circuit symbol

Theorems for exclusive OR

$X \oplus 0 = X$

$X \oplus 1 = X'$

$X \oplus X = 0$

$X \oplus X' = 1$

$X \oplus Y = Y \oplus X$

$(X \oplus Y) \oplus Z = X \oplus (Y \oplus Z) = X \oplus Y \oplus Z$

$X (Y \oplus Z) = XY \oplus XZ$

$(X \oplus Y)' = X \oplus Y' = X' \oplus Y = XY + X'Y'$

Equivalence (≡)

$0 \equiv 0 = 1$ $1 \equiv 0 = 0$

$0 \equiv 1 = 0$ $1 \equiv 1 = 1$

Truth table

X	Y	X ≡ Y
0	*0*	*1*
0	*1*	*0*
1	*0*	*0*
1	*1*	*1*

X ≡ Y = 1 iff X = Y

$X \equiv Y = XY + X'Y'$

Note: *Equivalence is the complement of Exclusive OR*

Circuit

Since complement of XOR

Ex. F = (A’B ≡ C) + (B ⊕ AC’)

A useful equation for manipulating an expression having many X-OR or equivalence operations

$$(XY' + X'Y)' = XY + X'Y'$$

Proof:

Ex. A’ ⊕ B ⊕ C

3.3 The Consensus Theorem

Given an expression in form XY + X'Z + YZ the YZ term is redundant and eliminated to form XY + X'Z, where YZ is known as the consensus term

Proof:

XY + X'Z + YZ =

Dual Form

(X+Y)(X'+Z)(Y+Z) = (X+Y)(X'+Z)

Ex. Dual (a + b + c') (a + b + d') (b + c + d')

Ex. A'C'D + A'BD + BCD + ABC + ACD'

Note: *Sometimes a term may need to be added first, using the consensus theorem, then use the added term to eliminate others.*

3.4 Algebraic Simplifications of Switching Expressions

Why? Reduction of costs

1) Combining Terms - *use XY + XY' = X to combine terms*

Ex. *abc'd' + abcd'*

X = abd' & Y = c

= abd'

To work you must have each side with one set of variable exact and another set as complements. This can be achieved by using X + X = X

Ex. ab'c + abc + a'bc (adding another abc)

ab'c + abc + a'bc + abc

ac + bc

2) Eliminating Terms - *use X + XY = X to eliminated redundant terms then try the consensus theorem (XY + X'Z + YZ = XY + X'Z) to eliminate excessive terms.*

3) Eliminate literals - *Use X + X'Y = X + Y to eliminate redundant literals*

4) Adding redundant terms - *Introduce redundant terms so they will combine to eliminate others.*

Add: XX'
YZ to XY + Y'Z
XY to X

Multiply by (X+X')

Ex. *A'B'C'D' + A'BC'D' + A'BD + A'BC'D + ABCD + ACD' + B'CD'*

3.5 Proving Validity of an Equation

Methods:

1. *Construct a truth table and evaluate both sides for all combinations of values of the variables.*

2. *Manipulate one side of the equation to equal the other using various theorems.*

3. *Reduce both sides independently to same expression.*

   You can perform the same operation to both sides as long as it is reversible

   Complementing both sides is permissible

   Multiplying or adding a number to each side is not because division and subtraction is undefined in Boolean algebra

When using 2 or 3 to prove validity a useful strategy is:

1. *Reduce both sides to sum of product or product of sums*

2. *Compare both sides to see how they differ*

3. *Try adding terms to one side that are present on the other*

4. *Try eliminating terms from one side that are not in the other*

Note: *To prove an equation is not valid, simply show one combination of values where the two sides have different values*

Ex. Proof: $A'BD' + BCD + ABC' + AB'D = BC'D' + AD + A'BC$

Review of Programmed Exercises 3.1 to 3.5 pages 68 – 73