Unit 5: Karnaugh Maps

5.1 Minimum Forms of Switching Functions

Minimum sum-of-products: a sum of product terms which; a) has a minimum number of terms and b) of all those expressions which have the same minimum number of terms, has a minimum number of literals.

To obtain a minimum sum-of-products from the minterm expansion:

1. Combine terms using XY' + XY = X. Repeat as much as possible. Remember X + X = X can be used to have a term appear more than once.

2. Eliminate redundant terms using the consensus theorem or other theorems.

Ex. Minimum sum-of-products

$$F(a, b, c) = \Sigma\, m(0, 1, 2, 5, 6, 7)$$

$$F = a'b'c' + a'b'c + a'bc' + ab'c + abc' + abc$$

$$F = a'b' + bc' + ac$$

Minimum product-of-sums: a product of sum terms which; a) has a minimum number of factors and b) of all those expressions which have the same minimum number of factors, has a minimum number of literals.

To obtain a minimum product-of-sums from the minterm expansion:

1. Combine terms using (X + Y')(X + Y) = X. Repeat as much as possible.

2. Eliminate redundant terms using the consensus theorem or other theorems.

Ex. Minimum product-of-sums

$$F = (A+B'+C+D')(A+B'+C'+D')(A+B'+C'+D)(A'+B'+C'+D)(A+B+C'+D)(A'+B+C'+D)$$

$$= (A + B' + D')(B' + C' + D)(B + C' + D)$$

$$= (A + B' + D')(C' + D)$$

5.2 Two and Three Variable Karnaugh Maps

Karnaugh Maps specify the values of a function for every combination of values of independent variables.

B \ A	0	1
0		
1		

Truth Table vs. Karnaugh Map

A	**B**	**F**
0	0	1
0	1	1
1	0	0
1	1	0

B \ A	0	1
0	1	0
1	1	0

Note: Each 1 in both the truth table and Karnaugh map represent a minterm.

In a Karnaugh map, adjacent minterms can be combined since they differ by only one variable (i.e. a'b' + a'b = a')

<u>3 Variables:</u>

A	**B**	**C**	**F**
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0

BC \ A	0	1
00	0	1
01	0	0
11*	1	0
10*	1	1

* The rows are written in sequence so adjacent rows differ by only one variable.

Again adjacent minterms differ by only one variable and can be reduced by XY'+XY = X.

Note: The top and bottom rows are also considered adjacent since they differ by only one variable.

<u>Minterm Expansion:</u> Given a minterm expansion plot a 1 in each square corresponding to a minterm.

$$F(A, B, C) = m_1 + m_3 + m_5$$

A	**B**	**C**	**F**
0	0	0	m_0
0	0	1	m_1
0	1	0	m_2
0	1	1	m_3
1	0	0	m_4
1	0	1	m_5
1	1	0	m_6
1	1	1	m_7

BC \ A	0	1
00	m_0	m_4
01	m_1	m_5
11*	m_3	m_7
10*	m_2	m_6

BC \ A	0	1
00	0	0
01	1	1
11*	1	0
10*	0	0

Note: For a maxterm expansion, 0's are plotted in the squares corresponding to the maxterms and the remaining squares are 1's.

<u>Product Terms:</u> a Karnaugh map can be created directly from product terms

f (a, b, c) = abc' + b'c + a'

1. Term abc' is 1 when a =1 and bc' = 10
2. Term b'c is 1 when b'c = 01 (2 locations)
3. Term a' is 1 when a = 0

bc \ a	0	1
00	1	
01	1	1
11	1	
10	1	1

Simplification:

bc \ a	0	1
00		
01	1	1
11	1	
10		

Minterms

001	101	011
1	5	3

f (a, b, c) = Σ m (1, 3, 5)

T_1 = a'b'c + a'bc

= a'c

T_2 = a'b'c + ab'c

= b'c

f = a'c + b'c

Complement: To map a complement of the function, replace the 0's with 1's and the 1's with 0's

Given: f (a, b, c) = Σ m (1, 3, 5)
f' = ?

bc \ a	0	1
00	1	1
01		
11		1
10	1	1

T_1 = a'b'c' + ab'c' + a'bc' + abc'

= b'c' + bc'

= c'

T_2 = abc + abc'

= ab

f' = c' + ab

<u>Consensus Theorem:</u> XY + X'Z + YZ = XY + X'Z

YZ \ X	0	1
00		
01	1	
11	1	1
10		1

YZ \ X	0	1
00		
01	1	
11	1	1
10		1

<u>Factors with Two Minimum Forms:</u>

bc \ a	0	1
00	1	
01	1	1
11		1
10	1	1

F = a'b' + ac +bc'

bc \ a	0	1
00	1	
01	1	1
11		1
10	1	1

F = a'c' + b'c + ab

5.3 Four Variable Karnaugh Maps

f (a, b, c, d) = acd + a'b + d'

CD \ AB	00	01	11	10
00	1	1	1	1
01		1		
11		1	1	1
10	1	1	1	1

Simplifying

CD \ AB	00	01	11	10
00		1	1	
01	1	1	1	
11	1			
10				1

$$F = T_1 + T_2 + T_3$$

$$F = bc' + a'b'd + ab'cd'$$

Incompletely Specified Functions:

When choosing terms, all 1’s must be circle but don’t cares are only used when helping to simplify.

CD \ AB	00	01	11	10
00			X	
01	1	1	X	1
11	1	1		
10		X		

$f = \Sigma m(1, 3, 5, 7, 9) + \Sigma d(6, 12, 13)$

$= c'd + a'd$

<u>Sum of Products:</u> Because the 0's of **f** are the 1's of **f'**, the minimum sum of products for **f'** can be found by looping the 0's of **f.** The complement of the minimum sum of products for **f'** is the minimum sum of products for **f**.

$f = x'z' + wyz + w'y'z' + x'y$

yz \ wx	00	01	11	10
00	1	1	0	1
01	0	0	0	0
11	1	0	1	1
10	1	0	0	1

$f' = y'z + wxz' + w'xy$

Minimum product of sums:

$(f')' = (y'z + wxz' + w'xy)'$

$f = (y'z)'(wxz')'(w'xy)'$

$f = (y + z')(w' + x' + z)(w + x' + y')$

5.4 Determination of Minimum Expressions Using Essential Prime Implicants

<u>Implicant:</u> a product term represented by any single 1 or group of 1's which can be combined together on a map of the function F

<u>Prime Implicant:</u> an implicant which cannot be combined with another term to eliminate a variable

All prime implicants can be obtained from a Karnaugh map

- All single 1's not adjacent to another 1
- Two adjacent 1's if not in a group of four
- Four adjacent 1's if not in a group of eight
- Etc.

cd \ ab	00	01	11	10
00	1		1	1
01			1	1
11	1			
10	1	1		

Prime implicants = a'b'c, a'cd', and ac'

Note: In order to find the minimum sum of products from a map, you must find the minimum number of prime implicant that cover all the 1' on the map.

cd \ ab	00	01	11	10
00		1	1	
01	1	1	1	
11	1		1	1
10			1	1

Minimum sum of products

f = a'b'd + bc' + ac

Prime implicants =

= a'b'd, bc', ac, a'c'd, ab, and b'cd

Note: In finding prime implicants don't cares are treated like 1's, however a prime implicant of all don't cares is never part of the minimum solution.

<u>Essential prime implicant:</u> a minterm that is only covered by one prime implicant and must be included in the minimum sum of products.

cd \ ab	00	01	11	10
00	1	1		
01	1	1		
11		1	1	1
10	1			

F = a'c' + a'b'd' + acd + (a'bd OR + bcd)

Procedure for finding the minimum sum of products from a Karnaugh Map

1. Choose a minterm (a 1) which has not yet been covered.

2. Find all 1’s and X’s adjacent to that minterm. (Check the *n* adjacent squares on an *n*-variable map.)

3. If a single term covers the minterm and all of the adjacent 1’s and X’s, then that term is an essential prime implicant, so select the term. (Note that don’t care terms are treated like 1’s in steps 2 and 3 but not step 1.)

4. Repeat steps 1, 2, and 3 until all essential prime implicants have been chosen.

5. Find a minimum set of prime implicants which cover the remaining 1’s on the map. (If there is more than one such set, choose a set with a minimum number of literals.)

See Figure 5-19 on page 132

5.5 Five Variable Karnaugh Maps

Each term can be adjacent to exactly five other terms, four in the same layer and one in the other layer.

5.6 Other Uses of Karnaugh Maps

<u>Minterm and Maxterm:</u> From a Karnaugh map, the minterm and maxterm expansion for an equation can be determined

<u>Proofs:</u> Two functions can be proven equal by plotting each on a map and showing that they have the sane Karnaugh map.

<u>Factoring:</u> By inspecting a Karnaugh map, you can reveal terms that have one or more variable in common

CD \ AB	00	01	11	10
00	1			
01	1			
11	1		1	
10			1	1

F = A'B'(C' + D) + AC(B +D')

5.7 Other Forms of Karnaugh Maps

<u>Veitch Diagrams</u>: Useful for plotting functions given algebraic forms rather than minterm/maxterm form

Alternate Forms for Five Variable Karnaugh Maps

DE \ BC	00	01	11	10
00	1	1	1	1
01	1		1	
11			1	
10				1

A = 0

DE \ BC	00	01	11	10
00	1	1	1	1
01	1		1	
11		1	1	
10				

A = 1

1	1	1	1
1		1	
		1	
			1

1	1	1	1
	1		1
	1	1	

F = D'E' + B'C'D' + BCE + A'BC'E' + ACDE