10.1 Instantaneous Power

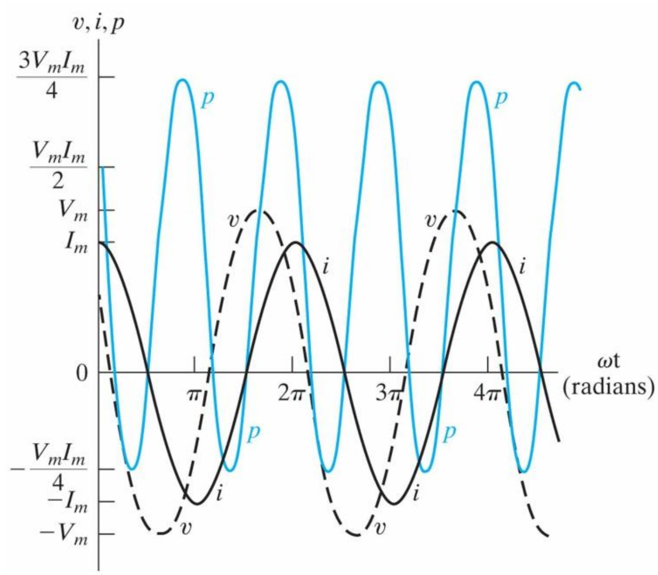
Instantaneous Power: product of the instantaneous terminal voltage and current; (positive when current is from positive to negative); the frequency of the power is twice the frequency of the voltage or current.

$$p = vi \text{ where } v = V_m \cos(\omega t + \theta_v) \text{ and } \quad i = I_m \cos(\omega t + \theta_i)$$

Utilizing trig. identities $\cos\alpha\cos\beta = \frac{1}{2}\cos(\alpha-\beta) + \frac{1}{2}\cos(\alpha+\beta)$ and

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$p = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i) + \frac{V_m I_m}{2}\cos(\theta_v + \theta_i)\cos 2\omega t - \frac{V_m I_m}{2}\sin(\theta_v - \theta_i)\sin 2\omega t$$



10.2 Average and Reactive Power

Average Power: the average value of the instantaneous power over one period; power converted from from electrical to nonelectrical form and vice versa: often referred to as the *real power*.

Reactive Power: electrical power exchanged between the magnetic field of an inductor and the source that drives it or between the electric field of a capacitor and the source that drives it. (Reactive power is never converted to nonelectric power)

Rewriting the three terms of the instantaneous power

$$p = P + P\cos 2\omega t - Q\sin 2\omega t$$

Where

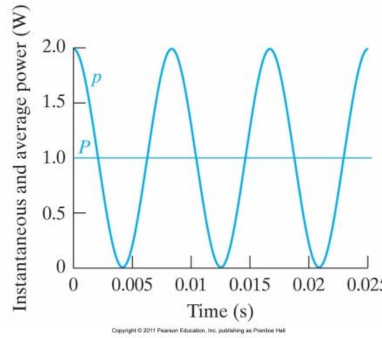
$$P = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i) = \text{ the average (real)power}$$

$$Q = \frac{V_m I_m}{2}\sin(\theta_v - \theta_i) \text{ the reactive power}$$

Purely resistive circuits: $\theta_v = \theta_i$

$$p = P + P\cos 2\omega t$$

Graphical representation assuming $\omega = 377\ \mathrm{rad/s}$

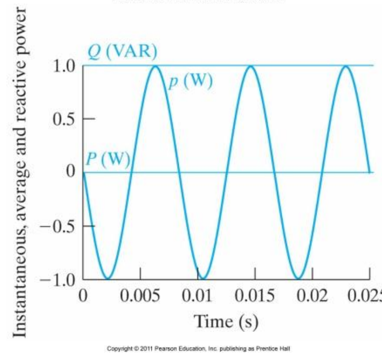


Purely inductive circuits: $\theta_i = \theta_v - 90°$
(Current lags voltage by 90°)

$$p = -Q\sin 2\omega t$$

Reactive power is given in units of *vars* (volt-amp reactive)

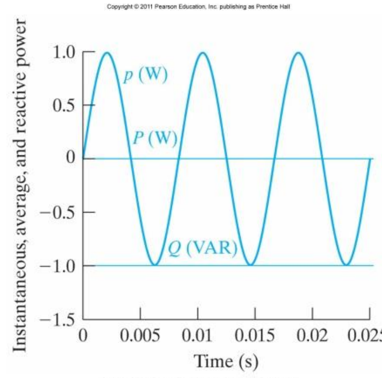
Graphical representation assuming $\omega = 377\ \mathrm{rad/s}$
(Note: average power is zero thus no energy transformation)



Purely capacitive circuits: $\theta_v - \theta_i = -90°$
(Current leads voltage by 90°)

$$p = -Q\sin 2\omega t$$

Graphical representation assuming $\omega = 377\ \mathrm{rad/s}$



Note: Inductors demand (absorb) vars; capacitor furnish (deliver) vars

<u>Power factor (pf):</u> cosine of the phase angle between the voltage and the current

$$\mathrm{pf} = \cos(\theta_v - \theta_i)$$

<u>Lagging power factor:</u> implies that the current lags the voltage – hence *inductive load*

<u>Leading power factor:</u> implies that the current leads the voltage – hence *capacitive load*

Reactive factor (rf): sine of the phase angle between the voltage and the current

$$\mathrm{rf} = \sin(\theta_v - \theta_i)$$

Review Example 10.1 & 10.2 and Assessment Problem 10.1 &10.2

10.3 The RMS Value of Power Calculations

From chapter 9: RMS is square root of the mean of the square of the function

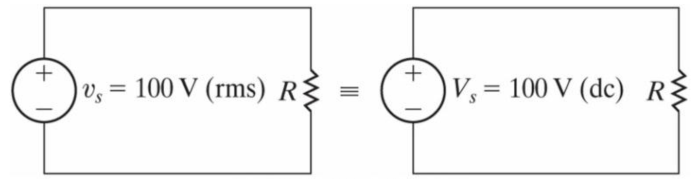
$$V_{rms} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} {V_m}^2 \cos^2(\omega t + \phi)\, dt} = \frac{\boldsymbol{V_m}}{\boldsymbol{\sqrt{2}}}$$

Average power can be written as

$$P = \frac{1}{T}\int_{t_0}^{t_0+T} \frac{{V_m}^2 \cos^2(\omega t + \phi)}{R} dt = \frac{V_{rms}^2}{R} = I_{rms}^2 R$$

<u>Effective value:</u> another name given to the RMS value

A DC source delivers the same energy over a given time as a sinusoidal source with the same rms value, assuming same load resistance.



Rewriting

$$P = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i) = \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}\cos(\theta_v - \theta_i) = V_{eff} I_{eff}\cos(\theta_v - \theta_i)$$

Therefore

$$Q = V_{eff} I_{eff}\sin(\theta_v - \theta_i)$$

Note: Effective values of sinusoids are widely used in rating equipment (i.e. 240/120V services)

Ex. The current used by a 100W light bulb in a 120V service (residential)

$$R = \frac{120^2}{100} = 144; \quad i = \frac{120}{144} = 0.833A\ rms \text{ or } 0.833\sqrt{2} = 1.18A\ peak$$

Review Example 10.3 and Assessment Problem 10.3

10.4 Complex Power

<u>Complex power (S):</u> sum of the real and reactive powers given in volt-amps (VA)

$$S = P + jQ$$

<u>Apparent Power:</u> magnitude of the complex power

$$|S| = \sqrt{P^2 + Q^2}$$

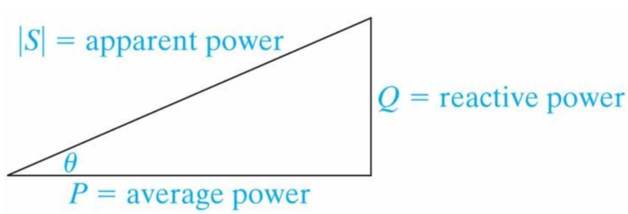


TABLE 10.2 Three Power Quantities and Their Units

Quantity	Units
Complex power	volt-amps
Average power	watts
Reactive power	var

Review Example 10.4

10.5 Power Calculations

Complex power

$$S = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i) + j\frac{V_m I_m}{2}\sin(\theta_v - \theta_i) = \frac{V_m I_m}{2}[\cos(\theta_v - \theta_i) + \text{j}\sin(\theta_v - \theta_i)]$$

$$= \frac{V_m I_m}{2}e^{j(\theta_v - \theta_i)} = \frac{1}{2}V_m I_m \angle(\theta_v - \theta_i)$$

RMS

$$S = V_{eff} I_{eff} \angle(\theta_v - \theta_i) = \overrightarrow{V_{eff}}\,\overrightarrow{I_{eff}}^*$$

Phasor

$$S = \frac{1}{2}\vec{V}\vec{I}^*$$

Given

$$\overrightarrow{V_{eff}} = Z\overrightarrow{I_{eff}}$$

$$S = Z\overrightarrow{I_{eff}}\,\overrightarrow{I_{eff}}^* = \left|\overrightarrow{I_{eff}}\right|^2 Z = P + jQ$$

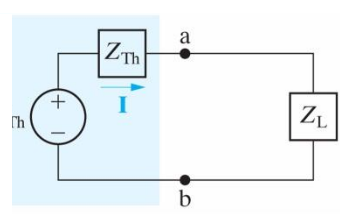
Therefore

$$P = \left|\overrightarrow{I_{eff}}\right|^2 R = \frac{1}{2}I_m^2 R = \frac{\left|\overrightarrow{V_{eff}}\right|^2}{R}$$

$$Q = \left|\overrightarrow{I_{eff}}\right|^2 X = \frac{1}{2}I_m^2 X = \frac{\left|\overrightarrow{V_{eff}}\right|^2}{X}$$

Review Example 10.5 – 10.7 and Assessment Problem 10.4 – 10.6

10.6 Maximum Power Transfer



$$Z_L = Z_{Th}^*$$

Maximum power

$$P_{max} = \left(\frac{|V_{Th}|}{2R_L}\right)^2 R_L = \frac{1}{4}\frac{|V_{Th}|^2}{R_L}$$

If voltage is not in RMS values then

$$P_{max} = \frac{1}{8}\frac{\boldsymbol{V}_m^2}{R_L}$$

Review Example 10.8 – 10.10 and Assessment Problem 10.7