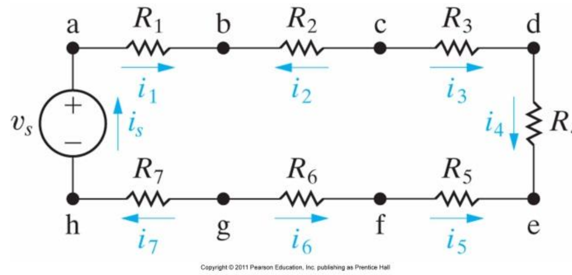
3.1 Resistors in Series

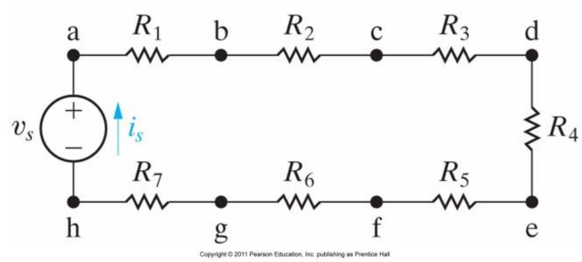
- Just Two elements connected at a single node are said to be in series.
- Series elements carry the same current



From the KCL

$$i_s = i_1 = -i_2 = i_3 = i_4 = -i_5 = -i_6 = i_7$$

From the KVL

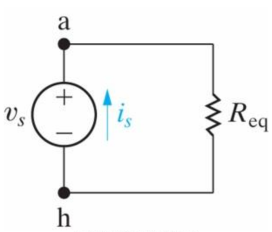


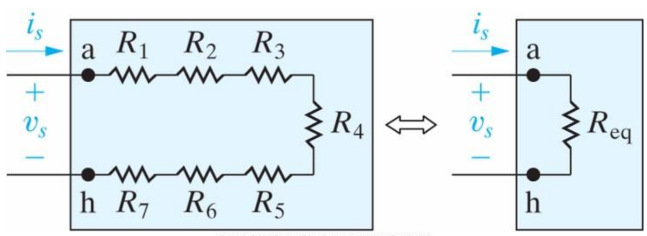
$$v_s = i_sR_1 + i_sR_2 + i_sR_3 + i_sR_4 + i_sR_5 + i_sR_6 + i_sR_7$$

$$v_s = i_s(R_1 + R_2 + R_3 + R_4 + R_5 + R_6 + R_7)$$

$$R_{eq} = R_1 + R_2 + R_3 + R_4 + R_5 + R_6 + R_7$$

$$\boldsymbol{v_s = i_sR_{eq}}$$





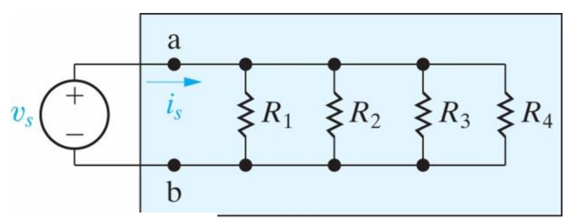
General equation for series resistance

$$R_{eq} = \sum_{i=1}^{k} R_i = R_1 + R_2 + \cdots + R_k$$

Note: The equivalent series resistance is *always larger* than the largest resistor in the series.

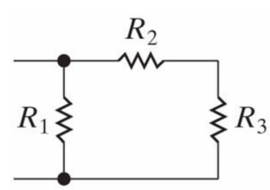
3.2 Resistors in Parallel

- Two elements connected at a single **node pair** are said to be in parallel.
- Parallel elements carry the same voltage across their terminals



From the KCL

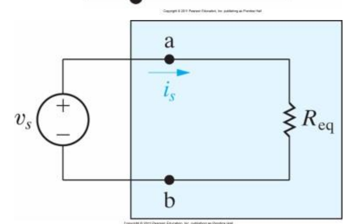
$$i_s = i_1 + i_2 + i_3 + i_4$$



From Ohm's Law

$$v_s = i_1R_1 = i_2R_2 = i_3R_3 = i_4R_4$$

Substituting and combining



$$\frac{i_s}{v_s} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} = \frac{1}{R_{eq}}$$

General equation for parallel resistance

$$\frac{1}{R_{eq}} = \sum_{i=1}^{k} \frac{1}{R_i} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_k}$$

Note: The equivalent parallel resistance is *always smaller* than the smallest resistor in the parallel connection.

Conductance equation for parallel resistance

$$G_{eq} = \sum_{i=1}^{k} G_i = G_1 + G_2 + \cdots + G_k$$

Two resistors in parallel

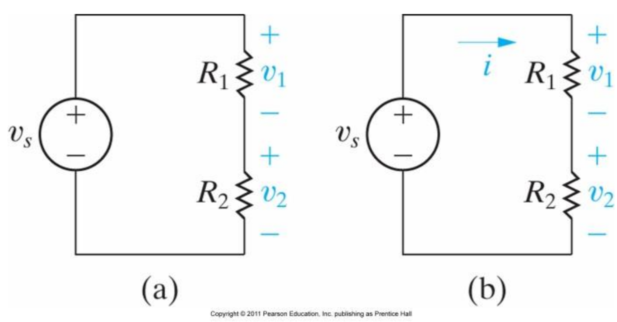
$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

Review Example Problem 3.1 and Assessment Problem 3.1

3.3 The Voltage-Divider and Current-Divider Circuits

The Voltage-divider circuit

KVL



$$v_s = iR_1 + iR_2$$

$$i = \frac{v_s}{R_1 + R_2}$$

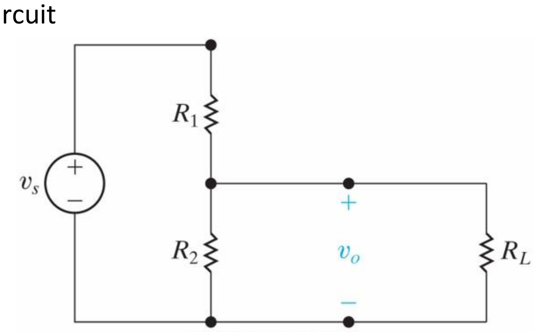
$$v_1 = iR_1 = \left(\frac{R_1}{R_1 + R_2}\right) v_s \qquad and \qquad v_2 = iR_2 = \left(\frac{R_2}{R_1 + R_2}\right) v_s$$

Note: to achieve the desired value for a voltage an infinite number of combinations of resistances exist thus other factors or design criteria will need to be used to determine the values.

Analyzing a circuit with a load

Load: one or more circuit elements that draw power from the circuit

"Load resistance" is in parallel with R_2

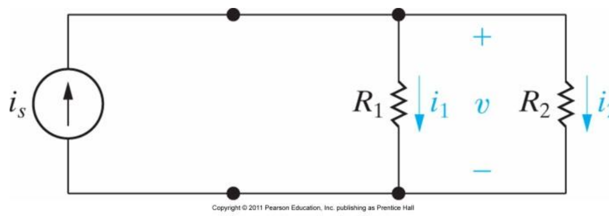


$$R_{eq} = \frac{R_2 R_L}{R_2 + R_L}$$

$$v_o = \left(\frac{R_{eq}}{R_1 + R_{eq}}\right) v_s = \left[\frac{R_2}{R_1\left(1 + {}^{R_2}/_{R_L}\right) + R_2}\right] v_s$$

Review Example 3.2

The Current-divider circuit



KCL $\qquad i_s = i_1 + i_2$

$$v = i_1 R_1 = i_2 R_2 = \left(\frac{R_1 R_2}{R_1 + R_2}\right) i_s$$

$$i_1 = \left(\frac{R_2}{R_1 + R_2}\right) i_s \qquad and \qquad i_2 = \left(\frac{R_1}{R_1 + R_2}\right) i_s$$

Review Examples 3.2 & 3.3 and Assessment Problems 3.2 & 3.3

3.4 Voltage Division and Current Division

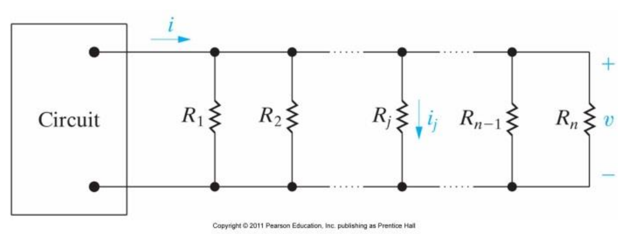
Voltage division: circuit analysis tool used to find the voltage drop across a single resistance from a collection of series resistances.

$$i = \frac{v}{R_1 + R_2 + \cdots + R_2} = \frac{v}{R_{eq}}$$

Voltage division equation

$$v_j = iR_j = \left(\frac{R_j}{R_{eq}}\right) v$$

Current division: circuit analysis tool used to find the current through a single resistance from a collection of parallel-connected resistances.



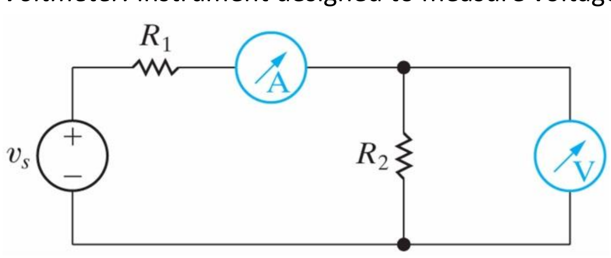
$$i_j = \frac{v}{R_j} = \left(\frac{R_{eq}}{R_j}\right) i$$

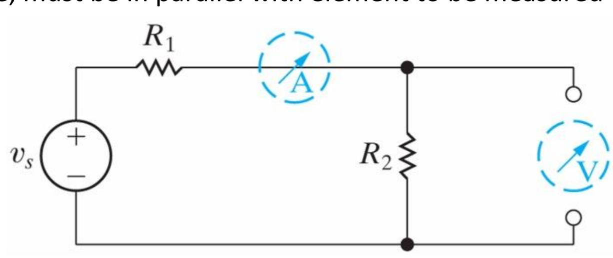
Review Example 3.4 and Assessment Problem 3.4

3.5 Measuring Voltage and Current

Ammeter: instrument designed to measure current; must be placed in sereies with element to be measured

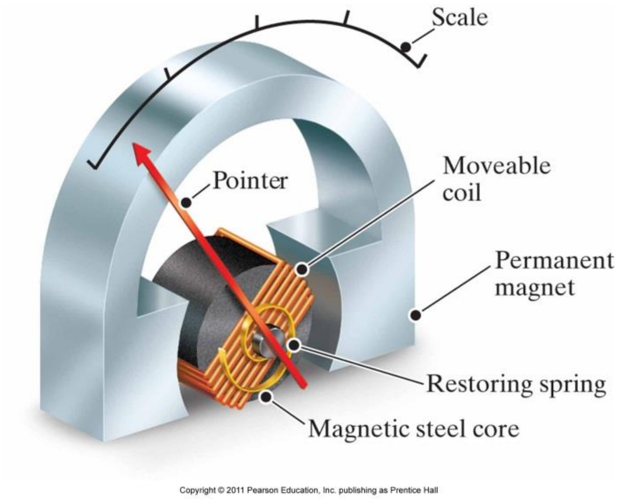
Voltmeter: instrument designed to measure voltage; must be in parallel with element to be measured

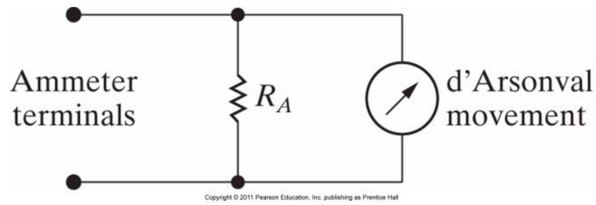


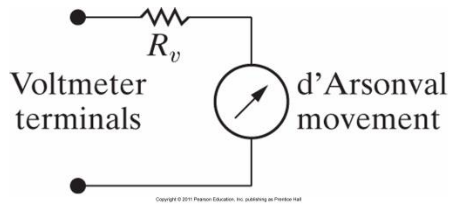


Digital meter: measures the continuous voltage or current signals at discrete points in time, the sample time

Analog meter: based on the d'Arsonval meter movement that implements a readout mechanism







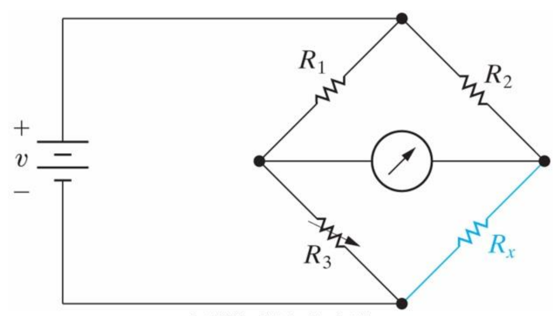
Review Examples 3.5 & 3.6 and Assessment Problems 3.5 & 3.6

3.6 Measuring Resistance - The Wheatstone Bridge

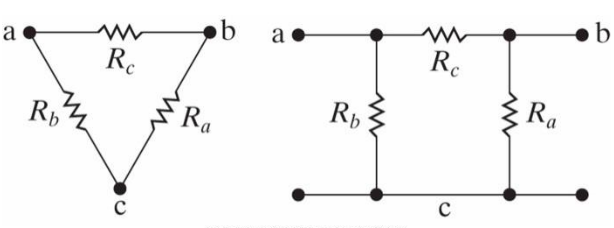
The variable resistor R_3 is varied to determine the value of the unknown R_x

$$R_x = \frac{R_2}{R_1} R_3$$

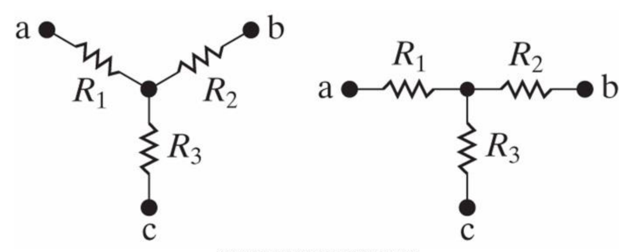
Review Assessment Problem 3.7



3.7 Delta-to-Wye (Pi to Tee) Equivalent Circuits



Delta Pi

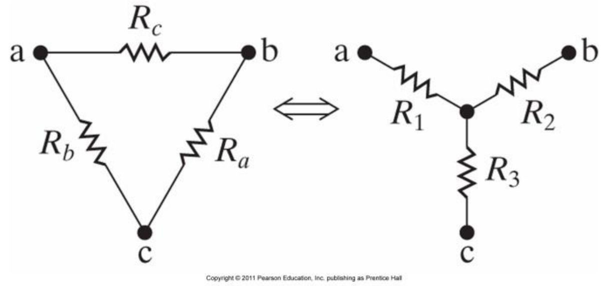


Wye Tee

A useful technique for circuit analysis is the ability to convert between delta and wye connected circuits. For this to be applicable the resistance between given nodes (i.e. a & b) must be the same for both circuits. Using the simplification tools for series and parallel resistance combinations a comparison can be made:

$$R_{ab} = \frac{R_c(R_a + R_b)}{R_a + R_b + R_c} = R_1 + R_2$$

For the delta connection R_{ab} is the combination of R_a in series with R_b then the combination in parallel with R_c.



$$R_{bc} = \frac{R_a(R_b + R_c)}{R_a + R_b + R_c} = R_2 + R_3$$

$$R_{ca} = \frac{R_b(R_c + R_a)}{R_a + R_b + R_c} = R_1 + R_3$$

Solving for the Delta to Wye (Δ-to-Y) conversion:

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c} \qquad R_2 = \frac{R_c R_a}{R_a + R_b + R_c} \qquad R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

Solving for the Wye to Delta (Y-to-Δ) conversion:

$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1} \qquad R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2} \qquad R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$$

Review Example 3.7 and Assessment Problem 3.8