Chapter 11: Balanced Three-Phase Circuits

11.1 Balanced Three-Phase Voltage

Comprised of three sinusoidal voltages identical in amplitude and frequency but out of phase from one another by 120°.

Referred to as **a-phase**, **b-phase** and c**-phase.**

Two Types of Phase Sequences

abc (positive) phase sequence

$$V_a = V_m\angle 0° \qquad V_b = V_m\angle -120° \qquad V_c = V_m\angle 120°$$

Phase b lags a by 120° and c leads a by 120°

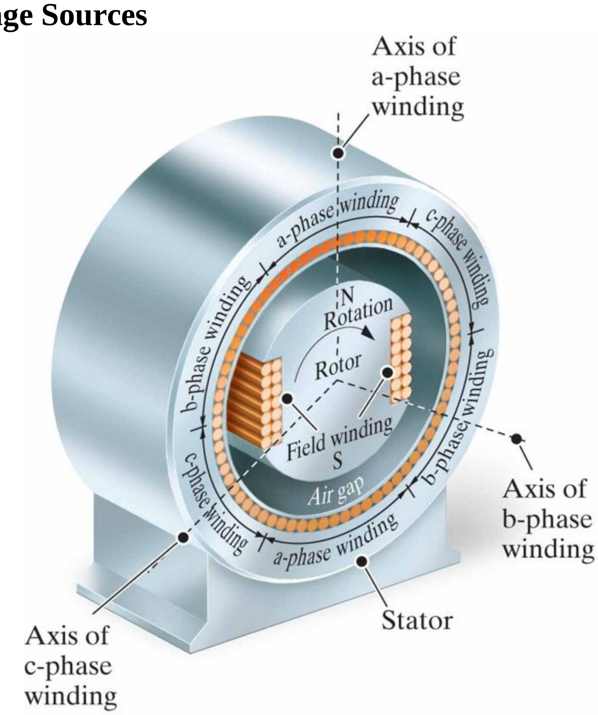
acb (negative) phase sequence

$$V_a = V_m\angle 0° \qquad V_b = V_m\angle 120° \qquad V_c = V_m\angle -120°$$

Phase c lags a by 120° and b leads a by 120°

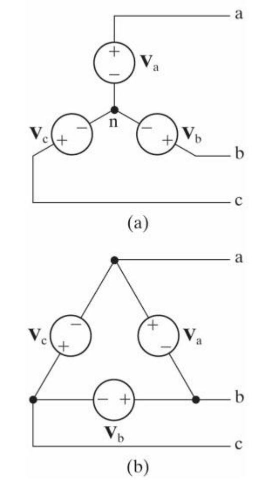
Important Characteristic $\quad V_a + V_b + V_c = 0 \quad and \quad v_a + v_b + v_c = 0$

11.2 Three-Phase Voltage Sources



A generator with three separate windings distributed around its stator, each winding comprising one phase. The rotor is an electromagnet driven at synchronous speed by a prime mover. The rotation induces sinusoidal voltages of equal amplitude and frequency that are out of phase 120° from one another.

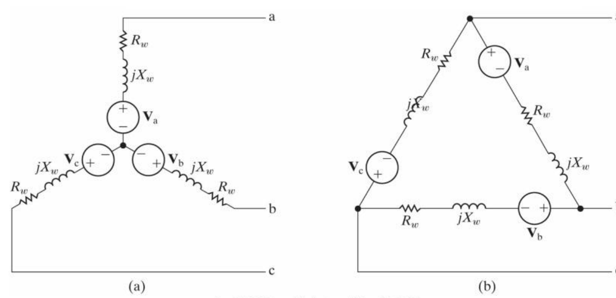
Two interconnection types:



Wye (Y)
The n-terminal is called the **neutral terminal**. (It may or may not be available for external connection)

Delta (Δ)

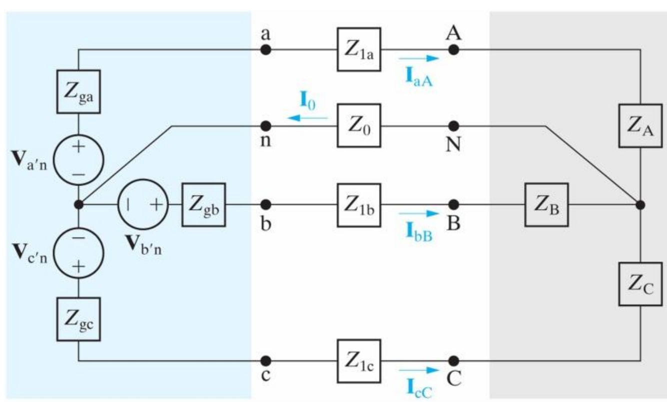
Since all windings are of the same construction the winding impedances are assumed to be identical.



Since 3-phase sources and loads can be connected either delta or wye there are four possible configurations:

Y – Y **Y – Δ** **Δ – Y** **Δ – Δ**

11.3 Analysis of the Wye-Wye Circuit



Z_{ga}, Z_{gb}, Z_{gc} Internal impedances

Z_{1a}, Z_{1b}, Z_{1c} Line impedances

Z_0 Impedance of the neutral

Z_A, Z_B, Z_C Load impedances

V_N Voltage between node N and n

General Equation – node voltage

$$\frac{\boldsymbol{V}_N}{Z_0} + \frac{\boldsymbol{V}_N - \boldsymbol{V}_{a'n}}{Z_A + Z_{1a} + Z_{ga}} + \frac{\boldsymbol{V}_N - \boldsymbol{V}_{b'n}}{Z_B + Z_{1b} + Z_{gb}} + \frac{\boldsymbol{V}_N - \boldsymbol{V}_{c'n}}{Z_C + Z_{1c} + Z_{gc}} = 0$$

Criteria for a balanced three-phased circuit

1. The voltage sources form a set of balanced three-phase voltages
2. The impedance of each phase of the voltage source are equal. $Z_{ga}, = Z_{gb} = Z_{gc}$.
3. The impedance of each line is the same. $Z_{1a} = Z_{1b} = Z_{1c}$.
4. The impedance of each phase load is equal. $Z_A = Z_B = Z_C$.

Rewriting the general equation based of the criteria

$$\boldsymbol{V}_N\left(\frac{1}{Z_0} + \frac{3}{Z_\varphi}\right) = \frac{\boldsymbol{V}_{a'n} + \boldsymbol{V}_{b'n} + \boldsymbol{V}_{c'n}}{Z_\varphi}$$

$$Z_{\varphi} = Z_A + Z_{1a} + Z_{ga} = Z_B + Z_{1b} + Z_{gb} = Z_C + Z_{1c} + Z_{gc}$$

According to the earlier assumption $\boldsymbol{V}_{a'n} + \boldsymbol{V}_{b'n} + \boldsymbol{V}_{c'n} = 0$ therefore

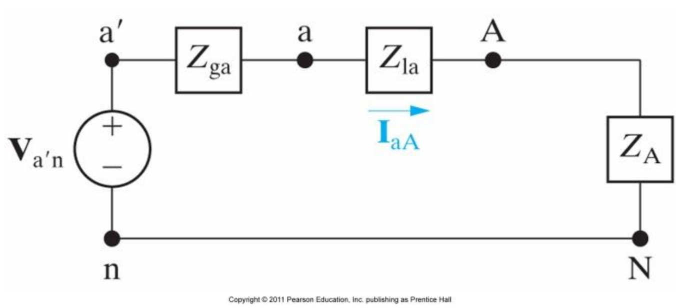
$$\boldsymbol{V}_N = 0$$

Balanced three-phase line currents

$$I_{aA} = \frac{\boldsymbol{V}_{a'n} - \boldsymbol{V}_N}{Z_A + Z_{1a} + Z_{ga}} = \frac{\boldsymbol{V}_{a'n}}{Z_{\varphi}}; \quad I_{bB} = \frac{\boldsymbol{V}_{bn} - \boldsymbol{V}_N}{Z_B + Z_{1b} + Z_{gb}} = \frac{\boldsymbol{V}_{b'n}}{Z_{\varphi}}; \quad I_{cC} = \frac{\boldsymbol{V}_{c'n} - \boldsymbol{V}_N}{Z_C + Z_{1c} + Z_{gc}} = \frac{\boldsymbol{V}_{c'n}}{Z_{\varphi}};$$

Notice the currents are equal in amplitude and frequency but are out of phase

Single-phase equivalent circuit:



Can be constructed as an equivalent circuit for the a-phase, with a shorted neutral, which represents the balanced three-phase circuit (The current in the neutral for the equivalent circuit is I_{aA}; which is not the same as in three-phase circuit)

Once the equivalent circuit is found, the current can be determined.

The values for the B and C phases can be determined from the A phase since they will have the same amplitude and frequency but are out of phase of A.

Once the current is known any of the voltage can be determined.

Line voltage: voltage across any pair of lines
Phase voltage: voltage across a single phase

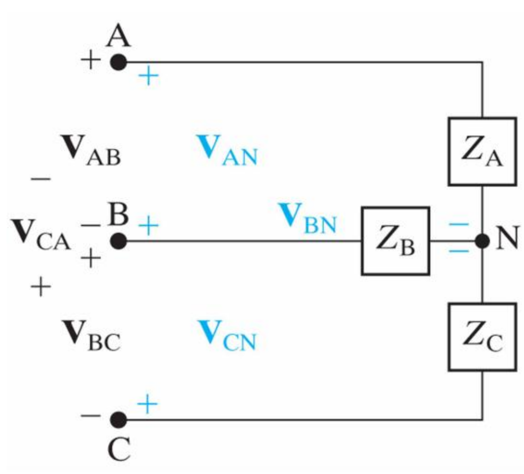
Line current: current in a single line
Phase current: current in a single phase

The *line–to–line voltage*s: the voltage drops from node to node

$$\boldsymbol{V}_{AB}, \boldsymbol{V}_{BC} \boldsymbol{V}_{CA}$$

The *line–to–neutral voltage*s: the voltage drops from node to neutral

$$\boldsymbol{V}_{AN}, \boldsymbol{V}_{BN} \boldsymbol{V}_{CN}$$



Relating the two voltages assuming positive sequence:

$$\boldsymbol{V}_{AB} = \boldsymbol{V}_{AN} - \boldsymbol{V}_{BN} \qquad \boldsymbol{V}_{BC} = \boldsymbol{V}_{BN} - \boldsymbol{V}_{CN} \qquad \boldsymbol{V}_{CA} = \boldsymbol{V}_{CN} - \boldsymbol{V}_{AN}$$

$$V_{AN} = V_{\varphi}\angle 0° \qquad V_{BN} = V_{\varphi}\angle -120° \qquad V_{CN} = V_{\varphi}\angle 120°$$

$$\boldsymbol{V}_{AB} = \sqrt{3}V_{\varphi}\angle 30° \qquad \boldsymbol{V}_{BC} = \sqrt{3}V_{\varphi}\angle -90° \qquad \boldsymbol{V}_{CA} = \sqrt{3}V_{\varphi}\angle 150°$$

11.4 Analysis of the Wye-Delta Circuit

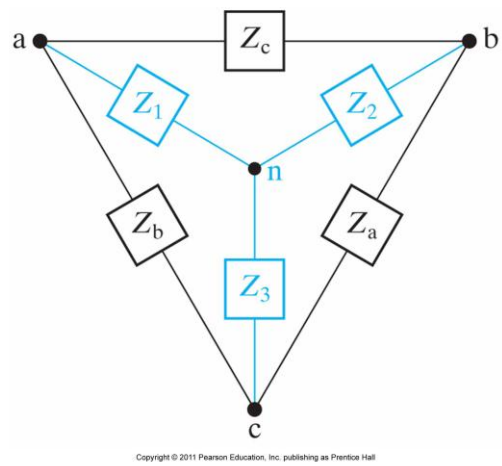
Option 1 Delta to Wye Transform

Review: (Chapter 9)

$$Z_1 = \frac{Z_b Z_c}{Z_a + Z_b + Z_c}$$

$$Z_2 = \frac{Z_c Z_a}{Z_a + Z_b + Z_c}$$

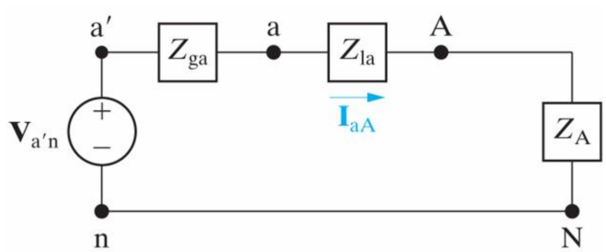
$$Z_3 = \frac{Z_a Z_b}{Z_a + Z_b + Z_c}$$



For a balanced three-phase system $Z_a = Z_b = Z_c$ therfore;

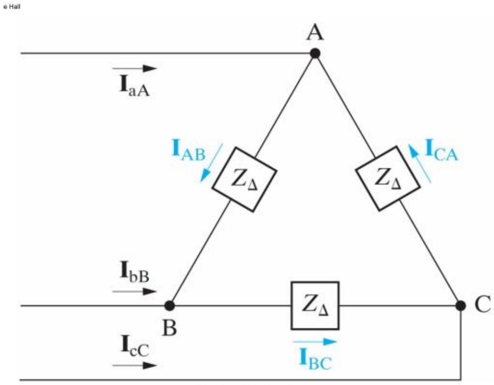
$$Z_Y = \frac{Z_\Delta}{3}$$

Then follow the techniques from the previous section by developing a single-phase equivalent circuit for a.



For a Delta load:

- The current in each leg is the phase current
- Voltage across each leg is the phase voltage
- Phase voltage is identical to line voltage



Assuming positive phase sequence and letting I_φ be the magnitude of the phase current:

$$I_{AB} = I_{\varphi}\angle 0° \qquad I_{BC} = I_{\varphi}\angle -120° \qquad I_{CA} = I_{\varphi}\angle 120°$$

Performing a KCL at the nodes

$$\boldsymbol{I}_{aA} = \boldsymbol{I}_{AB} - I_{CA} \qquad \boldsymbol{I}_{bB} = \boldsymbol{I}_{BC} - I_{AB} \qquad \boldsymbol{I}_{cC} = \boldsymbol{I}_{CA} - I_{BC}$$

$$\boldsymbol{I}_{aA} = \sqrt{3}I_{\varphi}\angle -30° \qquad \boldsymbol{I}_{bB} = \sqrt{3}I_{\varphi}\angle -150° \qquad \boldsymbol{I}_{cC} = \sqrt{3}I_{\varphi}\angle 90°$$

Comparing the two, the magnitude of the line is $\sqrt{3}$ larger than the phase and the line lags the phase by 30°. (negative sequence leads by 30°.)

11.5 Power Calculations in Balanced Three-Phase Circuits

Average Power in a Balanced Wye Load

Effective power $P = V_{eff}I_{eff}\cos(\theta_V - \theta_i)$ from chapter 10.3

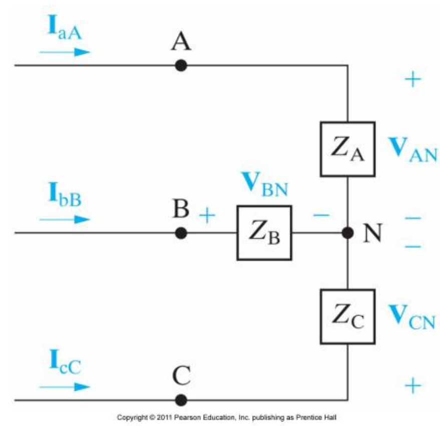
For a three-phase circuit (rms)

$$P_A = |V_{AN}||I_{aA}|\cos(\theta_{VA} - \theta_{iA})$$

Where $\theta_{VA}$and $\theta_{iA}$are phase angles of the voltage and current.

$$P_B = |V_{BN}||I_{bB}|\cos(\theta_{VB} - \theta_{iB})$$

$$P_C = |V_{CN}||I_{cC}|\cos(\theta_{VC} - \theta_{iC})$$



For a balanced load:

$$V_{\varphi} = |V_{AN}| = |V_{BN}| = |V_{CN}| \qquad I_{\varphi} = |I_{aA}| = |I_{bB}| = |I_{cC}|$$

$$\theta_{\varphi} = \theta_{VA} - \theta_{iA} = \theta_{VB} - \theta_{iB} = \theta_{VC} - \theta_{iC} \qquad P_{\varphi} = P_A = P_B = P_C = V_{\varphi}I_{\varphi}\cos\theta_{\varphi}$$

Total power delivered to the three-phase load $P_T = 3P_{\varphi}$

For line voltage V_L and current I_L in rms values $P_T = \sqrt{3}V_L\, I_L\, \cos\theta_{\varphi}$

Complex Power in a Balanced Wye Load

Reactive power Q= $V_{eff}I_{eff}\sin(\theta_V - \theta_i)$ from chapter 10.3

For a balanced load: $Q_{\varphi} = V_{\varphi}I_{\varphi}\sin\theta_{\varphi}$

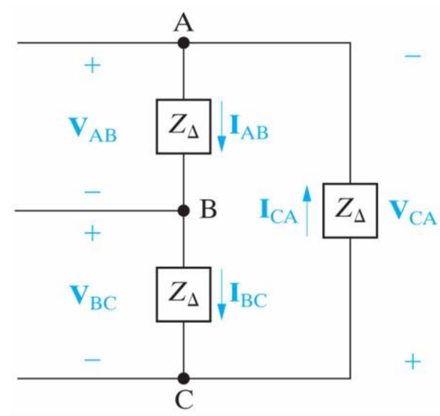
Total reactive power: $Q_T = 3Q_{\varphi} = \sqrt{3}V_L\, I_L\, \sin\theta_{\varphi}$

For complex power $S_{\varphi} = P_{\varphi} + jQ_{\varphi} = V_{\varphi}I_{\varphi}^*$

Total complex power: $S_T = 3S_{\varphi} = \sqrt{3}V_L\, I_L\, \angle\theta_{\varphi}^{\circ}$

Power Calculations in a Balanced Delta Load

The calculations are basically the same as the Wye



For a three-phase circuit (rms)

$$P_A = |V_{AB}||I_{AB}|\cos(\theta_{VAB} - \theta_{iAB})$$

$$P_B = |V_{BC}||I_{BC}|\cos(\theta_{VBC} - \theta_{iBC})$$

$$P_C = |V_{CA}||I_{CA}|\cos(\theta_{VCA} - \theta_{iCA})$$

For a balanced load:

$$V_\varphi = |V_{AB}| = |V_{BC}| = |V_{CA}| \qquad I_\varphi = |I_{AB}| = |I_{BC}| = |I_{CA}|$$

$$\theta_\varphi = \theta_{VAB} - \theta_{iAB} = \theta_{VBC} - \theta_{iBC} = \theta_{VCA} - \theta_{iCA}$$

$$P_\varphi = P_A = P_B = P_C = V_\varphi I_\varphi \cos\theta_\varphi$$

Total Power

$$P_T = \sqrt{3} V_L\, I_L\, \cos\theta_\varphi$$

$$Q_T = 3Q_\varphi = \sqrt{3} V_L\, I_L\, \sin\theta_\varphi$$

$$S_T = 3S_\varphi = \sqrt{3} V_L\, I_L\, \angle\theta_\varphi^\circ$$

Instantaneous Power in Three-Phase Circuits

$$p_A = v_{AN} i_{aA} = V_m I_m \cos\omega t \cos(\omega t - \theta_\varphi)$$

$$p_B = v_{BN} i_{bB} = V_m I_m \cos(\omega t - 120^\circ) \cos(\omega t - \theta_\varphi - 120^\circ)$$

$$p_C = v_{CN} i_{cC} = V_m I_m \cos(\omega t + 120^\circ) \cos(\omega t - \theta_\varphi + 120^\circ)$$

Total instantaneous power:

$$p_T = p_A = p_B = p_C = 1.5 V_m I_m \cos\theta_\varphi$$