Chapter 14: Introduction to Frequency Selective Circuits

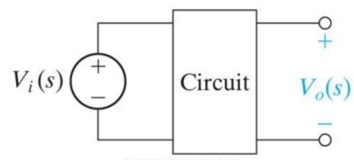
Filters: frequency selective circuits used in devices that communicate via electric signals like radio, phones, TV, etc. They have the ability to filter out certain input signals based on frequency.

Practical filters attenuate a signal rather than completely filter out

Attenuate: weaken or lessen the effects of

Passive filters: Filters that depend on passive circuit elements: resistors, capacitors and inductors. (Examined in this chapter)

14.1 Some Preliminaries



$$H(s) = \frac{V_{o(s)}}{V_i(s)}$$

Passband: signals passed from the input to the output

Stopband: signals not part of the passband

Frequency Response Plot: shows how the transfer function changes as the source frequency changes

2-parts
Magnitude plot: graph of $|H(j\omega)|$ versus ω

Phase plot: graph of $\theta(j\omega)$ versus ω

4 Types of Filters

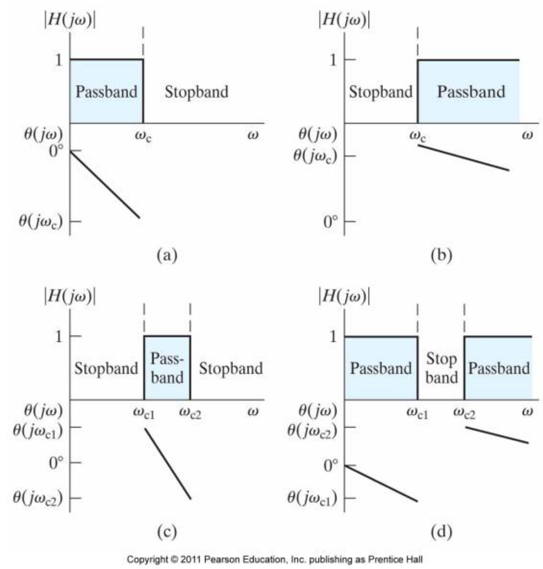
1. Low-pass: Have one passband and one stopband which are characterized by the cutoff frequency. Passes all frequencies **lower** than the cutoff frequency from input to output

2. High-pass: Have one passband and one stopband which are characterized by the cutoff frequency. Passes all frequencies **higher** than the cutoff frequency from input to output

3. Bandpass: Have two cutoff frequencies and passes input frequencies to the output only when they are between the two cutoff frequencies.

4. Bandreject: Have two cutoff frequencies and passes input frequencies to the output only when they are outside of the two cutoff frequencies.

Figure 14.3 Ideal frequency response plots of the four types of filter circuits. (a) An ideal low-pass filter. (b) An ideal high-pass filter. (c) An ideal bandpass filter. (d) An ideal bandreject filter.

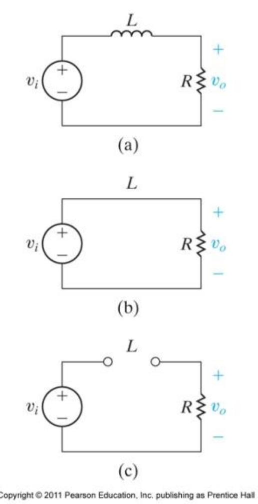


Note: Phase angle plots for ideal filters vary linearly in the passband and are irrelevant elsewhere. This linear phase variation is necessary to avoid phase distortion.

14.2 Low-Pass Filters

<u>Series RL Circuit – Qualitative Analysis</u>

$$Given\ \ v_i = sinusoid\ with\ varying\ frequency$$
$$where\ Z_L = j\omega L$$

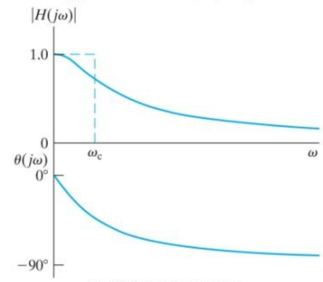


At low frequencies $\omega L \ll R$; the inductor appears as a short and therefore the circuit can be modeled as such with $\omega = 0$. There is no change in magnitude or phase from input to output

As the frequency increases, the inductor impedance increases and thus introduces a phase shift between the input and output.

At high frequencies $\omega L \gg R$; the inductor appears as an open circuit and therefore the circuit can be modeled as such with $\omega = \infty$. The magnitude of the output is 0 and the phase is -90

Thus the high frequencies are blocked



By observing a magnitude and phase plot of an actual RL low-pass filter it can be seen that the magnitude changes slowly from the passband to stopband unlike an ideal.

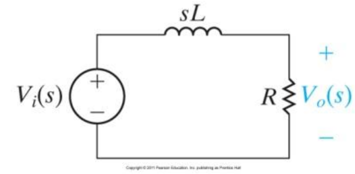
It is therefore necessary to define the cutoff frequency for a real circuit

Defining the Cutoff Frequency:

$$|H(j\omega_c)| = \frac{1}{\sqrt{2}} H_{max}$$

Where ω_c is also called the half power frequency because in the passband the average power delivered to the load is at least 50% of the maximum average power

<u>Series RL Circuit – Quantitative Analysis</u>



$Transfer\ Function$: $$H(s) = \frac{R/L}{s + R/L}$$

For $s = j\omega$

$$H(j\omega) = \frac{R/L}{j\omega + R/L}$$

Separating into the function into magnitude and phase components;

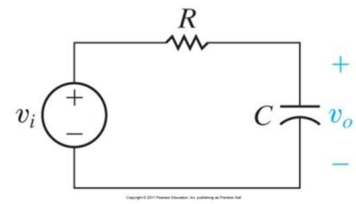
$$|H(j\omega)| = \frac{R/L}{\sqrt{\omega^2 + \left(R/L\right)^2}} \quad and \quad \theta(j\omega) = -\tan^{-1}\left(\frac{\omega L}{R}\right)$$

From the magnitude equation

$$|H(j\omega_c)| = \frac{1}{\sqrt{2}} H_{max} = \frac{1}{\sqrt{2}} 1 = \frac{R/L}{\sqrt{\omega^2 + \left(R/L\right)^2}}$$

$Therefore$ $$\boldsymbol{\omega_c = \frac{R}{L}}$$

<u>A Series RC Circuit</u>:



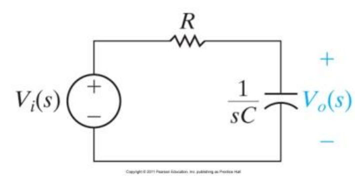
Qualitative analysis

1. Zero frequency: the impedance of the capacitor is infinite thus acting like an open circuit. The input and output voltages are the same.

2. Frequencies increasing from zero: the impedance of the capacitor decreases relative to the resistor. The output voltage is smaller than the source.

3. Infinite frequency: the impedance is zero and therefore the output voltage is zero.

Example 14.2



$$H(s) = \frac{1/RC}{s + 1/RC} \quad thus \quad H(j\omega) = \frac{1/RC}{\sqrt{\omega^2 + \left(1/RC\right)^2}}$$

$$|H(j\omega_c)| = \frac{1}{\sqrt{2}}1 = \frac{1/_{RC}}{\sqrt{\omega^2 + \left(1/_{RC}\right)^2}}$$

$$\omega_c = \frac{1}{RC}$$

General low-pass filter transfer function

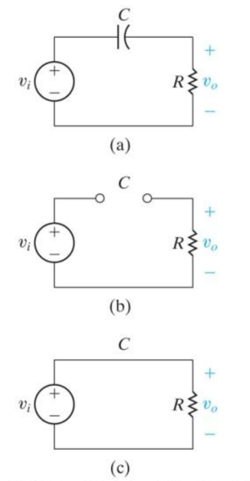
$$H(s) = \frac{\omega_c}{s + \omega_c}$$

Relating frequency to time domain

$$\tau = \frac{L}{R} = RC = \frac{1}{\omega_c}$$

14.3 High-Pass Filters

A Series RC Circuit - Qualitative Analysis



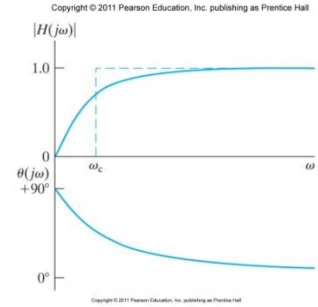
The output for this circuit is across the resistor; whereas it was the capacitor in the low-pass filter.

At $\omega = 0$ the capacitor acts as an open circuit and no current flows through the resistor thus there is no voltage across the resistor. The input does not reach the output.

As the frequency increases, the capacitor's impedance decreases and thus the magnitude of the output voltage increases.

At $\omega = \infty$ the capacitor acts as a short circuit the input and output voltage are the same.

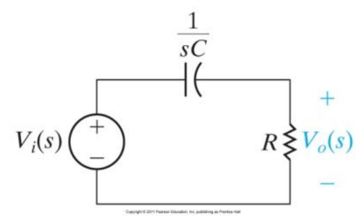
Thus the low frequencies are blocked



By observing a magnitude and phase plot of a high-pass filter the phase angle can be seen to vary from 0 at $\omega = \infty$ to +90 at $\omega = 0$

Note: The high-pass filter circuit is identical to the low-pass filer the only difference is the choice of the output.

Series RC Circuit – Quantitative Analysis



$$H(s) = \frac{s}{s + 1/_{RC}} \quad thus \quad H(j\omega) = \frac{j\omega}{j\omega + 1/_{RC}}$$

Solving for the magnitude and phase:

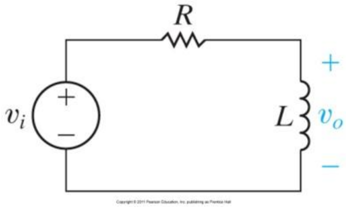
$$|H(j\omega_c)| = \frac{\omega}{\sqrt{\omega^2 + \left(1/_{RC}\right)^2}}$$

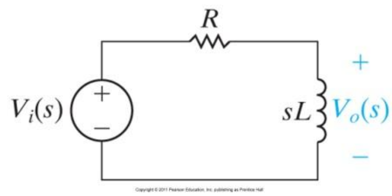
$$\theta(j\omega) = 90° - \tan^{-1}(\omega RC)$$

$$\omega_c = \frac{1}{RC}$$

Note: The cutoff frequency for an RC circuit is $1/_{RC}$ for both the high-pass and low-pass filters.

Example 14.3 – RL High-pass circuit





$$H(s) = \frac{s}{s + {}^{R}/_{L}} \quad thus \quad H(j\omega) = \frac{j\omega}{j\omega + {}^{R}/_{L}}$$

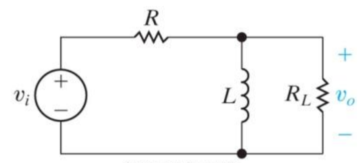
$$|H(j\omega_c)| = \frac{\omega}{\sqrt{\omega^2 + \left({}^{R}/_{L}\right)^2}} \quad and \quad \omega_c = \frac{R}{L}$$

Note: The cutoff frequency is the same as the low-pass RL filter.

To determine the values of R and L it is necessary to choose one of the values (a common value) and solve for the other

Example 14.4 – Loaded RL High-pass circuit

$$H(s) = \frac{\frac{R_L sL}{R_L + sL}}{R + \frac{R_L sL}{R_L + sL}} = \frac{\left(\frac{R_L}{R + R_L}\right)s}{s + \left(\frac{R_L}{R + R_L}\right)\frac{R}{L}} = \frac{Ks}{s + \omega_c}$$



Where

$$K = \left(\frac{R_L}{R + R_L}\right) \; and \; \omega_c = K\frac{R}{L}$$

Effects of loading on a filter

- The largest amplitude possible for a passive filter is 1and adding a load resistance only further reduces it.
- For design purposes we normally want the filter's transfer function to remain the same regardless of loading which is not the case for passive filters.

Active filters which will be discussed in the next chapter will allow us to overcome these issues.

General High-pass filter transfer function

$$H(s) = \frac{s}{s + \omega_c}$$

14.4 Bandpass Filters

5 Parameters of a Bandpass Filter

- The two **cut-off frequencies** ω_{c1} *and* ω_{c2}

- **Center frequency** (ω_o): the frequency at which the transfer function is entirely real. Aka the resonant frequency

  Located at the geometric center of the passband

  $$\omega_o = \sqrt{\omega_{c1}\ \omega_{c2}}$$

  The magnitude of the function is maximum at the center frequency

  $$H_{max} = |H(j\omega_o)|$$

- **Bandwidth** (β): width of the passband

  $$\beta = \omega_{c2} - \omega_{c1}$$

- **Quality Factor** (Q): ratio of the center frequency to the bandwidth

  $$Q = \frac{\omega_o}{\beta}$$

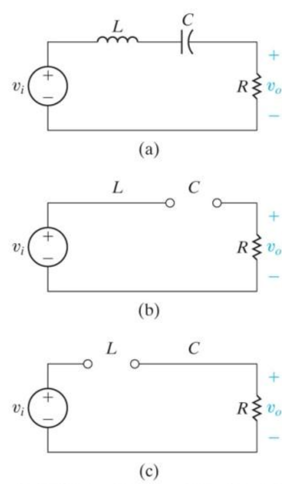
Note: Knowing any two of the above parameters allows one to calculate the remaining

<u>Series RLC Circuit – Qualitative Analysis</u>

At $\omega = 0$ the capacitor acts as an open circuit the inductor acts as a short and no current flows through the resistor thus there is no voltage across the resistor.

At $\omega = \infty$ the capacitor acts as a short circuit but the inductor acts as an open circuit and no current flows through the resistor thus there is no voltage across the resistor.

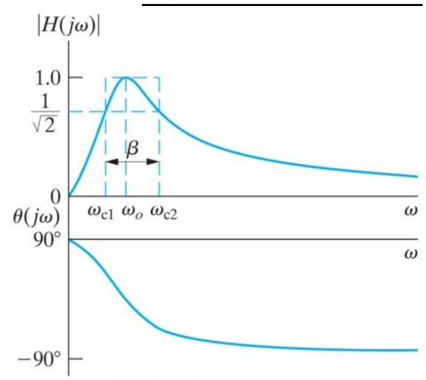
Between $\omega = 0$ and $\omega = \infty$, the capacitor and inductor have finite impedance values thus there is some output voltage. (Capacitor impedance is negative, inductor positive)



At the frequency where the capacitor and inductor impedances are equal, but opposite, they cancel and V_o will be max (center frequency).

At the center frequency the series combination of the capacitor and inductor is a short.

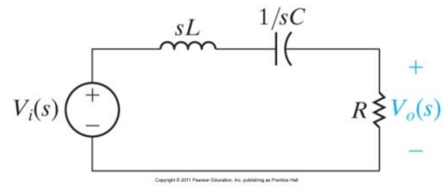
Looking at a magnitude and phase plot



The magnitude reach the max, 1, at the center frequency and has cut-offs on either side

The phase is +90 at zero fall to zero at the center frequency continuing to -90 at infinity

Series RLC Circuit – Quantitative Analysis



$$H(s) = \frac{\frac{R}{L}s}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

$$|H(j\omega_c)| = \frac{\omega\frac{R}{L}}{\sqrt{\left(\frac{1}{LC} - \omega^2\right)^2 + \left(\omega\, {}^{R}/_{L}\right)^2}} \quad and \quad \theta(j\omega) = 90° - \tan^{-1}\left(\frac{\omega\frac{R}{L}}{\frac{1}{LC} - \omega^2}\right)$$

Since $\omega_o$is where the function is totally real;
that is where $j\omega_o L + \frac{1}{j\omega_o C} = 0$

$$\omega_o = \sqrt{{}^{1}/_{LC}}$$

The cut-off frequencies are at $\frac{1}{\sqrt{2}}H_{max}$

$$\frac{1}{\sqrt{2}} = \frac{\omega_c\frac{R}{L}}{\sqrt{\left(\frac{1}{LC} - \omega_c^2\right)^2 + \left(\omega_c\, {}^{R}/_{L}\right)^2}} = \frac{1}{\sqrt{\left(\frac{\omega_c L}{R} - \frac{1}{\omega_c RC}\right)^2 + 1}}$$

Setting the denominator equal

$$\sqrt{2} = \sqrt{\left(\frac{\omega_c L}{R} - \frac{1}{\omega_c RC}\right)^2 + 1}$$

Rewriting

$$\frac{\omega_c L}{R} - \frac{1}{\omega_c RC} = \pm 1$$

Solving with quadratic yields

$$\omega_{c1} = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

$$\omega_{c2} = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

The bandwidth:

$$\beta = \omega_{c2} - \omega_{c1} = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} - \left(-\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}\right) = \frac{R}{L}$$

Quality Factor:

$$Q = \frac{\omega_o}{\beta} = \frac{\sqrt{{}^{1}/_{LC}}}{{}^{R}/_{L}} = \sqrt{\frac{L}{CR^2}}$$

Cut-off frequencies in terms of bandwidth and center frequency

$$\omega_{c1} = -\frac{\beta}{2} + \sqrt{\left(\frac{\beta}{2}\right)^2 + \omega_o^2}$$

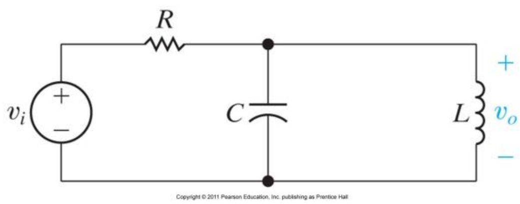
$$\omega_{c2} = \frac{\beta}{2} + \sqrt{\left(\frac{\beta}{2}\right)^2 + \omega_o^2}$$

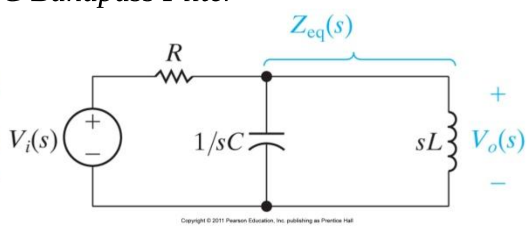
Cut-off frequencies in terms of quality factor and center frequency

$$\omega_{c1} = \omega_o\left(-\frac{1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q}\right)^2}\right)$$

$$\omega_{c2} = \omega_o\left(\frac{1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q}\right)^2}\right)$$

Example 14.6 – Designing a Parallel RLC Bandpass Filter





Finding Z_{eq} then the transfer function

$$Z_{eq}(s) = \frac{\frac{L}{C}}{sL + \frac{1}{sc}} \quad therefore \quad H(s) = \frac{\frac{s}{RC}}{s^2 + \frac{s}{RC} + \frac{1}{LC}}$$

Again, the cut-off frequencies are found from the magnitude

$$|H(j\omega_c)| = \frac{\frac{\omega_c}{RC}}{\sqrt{\left(\frac{1}{LC} - \omega_c^2\right)^2 + \left(\frac{\omega_c}{RC}\right)^2}} = \frac{1}{\sqrt{\left(\frac{1}{LC} - \omega_c^2\right)^2 + \left(\frac{\omega_c}{RC}\right)^2}} = \frac{1}{\sqrt{2}}$$

$$\omega_{c1} = -\frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}; \quad \omega_{c2} = \frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$$

$$\beta = \omega_{c2} - \omega_{c1} = \frac{1}{RC}$$

$$Q = \frac{\omega_o}{\beta} = \sqrt{\frac{CR^2}{L}}$$

General Bandpass filter transfer function

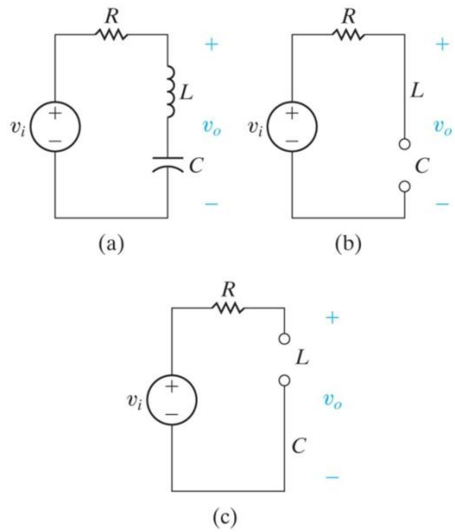
$$H(s) = \frac{\beta s}{s^2 + \beta s + \omega_o^2}$$

Relating frequency to time domain

$$\alpha = \frac{R}{2L}; \quad \omega_0 = \sqrt{\frac{1}{LC}}; \quad \beta = 2\alpha$$

14.5 Bandreject Filters

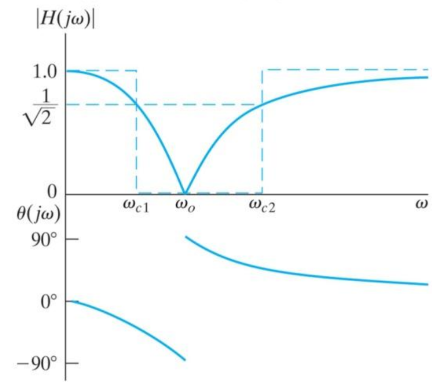
Series RLC Circuit – Qualitative Analysis



Note: The output voltage is now defined across the inductor-capacitor pair.

Again, at $\omega = 0$ the capacitor acts as an open circuit the inductor acts as a short and at $\omega = \infty$ the capacitor acts as a short circuit but the inductor acts as an open circuit. The voltage is across an effective open circuit and the output equals the input.

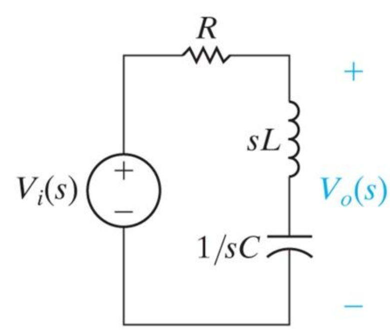
Between $\omega = 0$ and $\omega = \infty$, the capacitor and inductor have finite impedance values reducing the output voltage and at a certain are equal and opposite resulting in no voltage output.



The magnitude frequency response plot compares the ideal Bandreject filter to the actual response seen from the RLC Circuit.

The effects of the phase shift due to the capacitor and inductor in the phase place. Starting from zero the phase gets more negative till reaching -90 at which point it flips to +90 and goes negative again towards zero.

Series RLC Circuit – Quantitative Analysis



Using voltage division

$$H(s) = \frac{sL + \frac{1}{sC}}{R + sL + \frac{1}{sC}} = \frac{s^2 + \frac{1}{LC}}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

Magnitude & phase

$$|H(j\omega_c)| = \frac{\left|\frac{1}{LC} - \omega^2\right|}{\sqrt{\left(\frac{1}{LC} - \omega^2\right)^2 + \left(\frac{\omega R}{L}\right)^2}}$$

$$\theta(j\omega) = -\tan^{-1}\left(\frac{\omega \frac{R}{L}}{\frac{1}{LC} - \omega^2}\right)$$

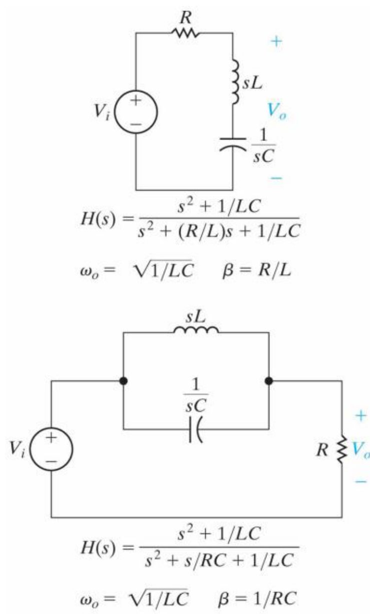
The five parameters for the Bandreject are the same as the Bandpass

$$\omega_o = \sqrt{1/_{LC}}; \quad \omega_{c1} = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}; \ \omega_{c2} = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

$$\beta = \omega_{c2} - \omega_{c1} = \frac{R}{L} \qquad Q = \sqrt{\frac{L}{CR^2}}$$

The alternate forms of the cutoff frequencies in terms of bandwidth and quality factor would also remain the same.

Figure 14.31 Two *RLC* bandreject filters, together with equations for the transfer function, center frequency, and bandwidth of each.



General Bandreject filter transfer function

$$H(s) = \frac{s^2 + \omega_o^2}{s^2 + \beta s + \omega_o^2}$$