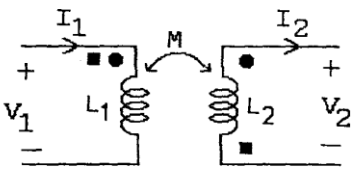
Mutual Inductance and Linear Transformers



Round dots:

$$\vec{V}_1 = j\omega L_1 \vec{I}_1 - j\omega M \vec{I}_2$$
$$\vec{V}_2 = +j\omega M \vec{I}_1 - j\omega L_2 \vec{I}_2$$

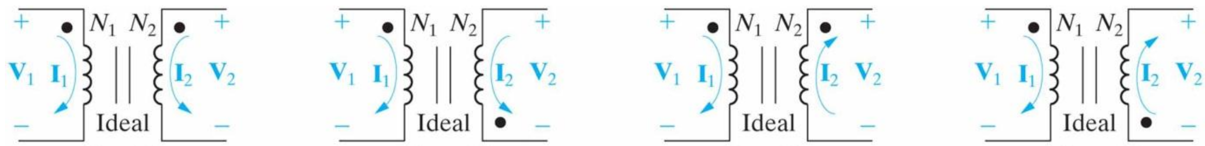
Square dots:

$$\vec{V}_1 = j\omega L_1 \vec{I}_1 + j\omega M \vec{I}_2$$
$$\vec{V}_2 = -j\omega M \vec{I}_1 - j\omega L_2 \vec{I}_2$$

Reflected impedance $Z_r = \frac{\omega^2 M^2}{|Z_{22}|^2}[(R_2 + R_L) - j(\omega L_2 + X_L)]$ where $Z_{22} = R_2 + R_L + j(\omega L_2 + X_L)$

$$\boldsymbol{M = k\sqrt{L_1 L_2}} \qquad \boldsymbol{w(t) = \frac{L_1 i_1^2}{2} + \frac{L_2 i_2^2}{2} \pm i_1 i_2 M}$$

Ideal Transformers



(a) $\frac{\mathbf{V}_1}{N_1} = \frac{\mathbf{V}_2}{N_2}$, $N_1\mathbf{I}_1 = -N_2\mathbf{I}_2$

(b) $\frac{\mathbf{V}_1}{N_1} = -\frac{\mathbf{V}_2}{N_2}$, $N_1\mathbf{I}_1 = N_2\mathbf{I}_2$

(c) $\frac{\mathbf{V}_1}{N_1} = \frac{\mathbf{V}_2}{N_2}$, $N_1\mathbf{I}_1 = N_2\mathbf{I}_2$

(d) $\frac{\mathbf{V}_1}{N_1} = -\frac{\mathbf{V}_2}{N_2}$, $N_1\mathbf{I}_1 = -N_2\mathbf{I}_2$

The impedance seen by the source $Z_{in} = \frac{\boldsymbol{V}_1}{\boldsymbol{I}_1} = \frac{1}{a^2}\frac{\boldsymbol{V}_2}{\boldsymbol{I}_2} = \frac{1}{a^2} Z_L$ where $a = \frac{N_2}{N_1}$

AC Information

$v = V_m \cos(\omega t + \phi) \qquad f = \frac{1}{T} = \frac{\omega}{2\pi}$

$\boldsymbol{V} = Z\boldsymbol{I} \qquad \overrightarrow{V_{eff}} = Z\overrightarrow{I_{eff}}$

Rectangular to Polar conversion

$$\boldsymbol{V} = A + jB = \sqrt{A^2 + B^2}\angle\left(\tan^{-1}\frac{B}{A}\right)^\circ$$

Polar to Rectangular conversion

$$\boldsymbol{V} = V_m\angle\phi^\circ = V_m \cos\phi + jV_m \sin\phi$$

(Note that V_m is a magnitude and should be positive, thus angle needs to reflect the change)

Complex Number Identities

$$j^2 = -1 \qquad \text{and} \qquad \frac{1}{j} = -j$$

<u>Lagging power factor:</u> implies that the current lags the voltage – hence *inductive load*

<u>Leading power factor:</u> implies that the current leads the voltage – hence *capacitive load*

Quantity	Symbol	Unit	Formulas
Complex power	S	VA	$S = \vec{V}\vec{I}^*$ $= V_{rms}I_{rms}\angle\theta$ $= P + jQ = I_{rms}^2 Z$
Apparent power	$\|S\|$	VA	$\|S\| = V_{rms}I_{rms}$ $= \sqrt{P^2 + Q^2}$
Average power	P	W	$P = \mathrm{Re}\{S\}$ $= V_{rms}I_{rms}\cos\theta$ $= I_{rms}^2\,\mathrm{Re}\{Z\}$
Reactive power	Q	var	$Q = \mathrm{Im}\{S\}$ $= V_{rms}I_{rms}\sin\theta$ $= I_{rms}^2\,\mathrm{Im}\{Z\}$
Instantaneous power	p(t)	W	$p(t) = P + P\cos 2\omega t + Q\sin 2\omega t$
Power factor angle	θ	deg	$\theta = \angle S = \angle Z$ $= \angle\vec{V} - \angle\vec{I}$
Power factor	pf	Lagging if θ>0 & Q>0; leading if θ<0 & Q<0.	$pf = \frac{P}{\|S\|}$ $= \cos\theta$

BALANCED THREE-PHASE SYSTEMS:

Balanced source: equal magnitudes; phase angle separated by 120 deg

Balanced load or line: equal phase impedances

Balanced system = balanced source + balanced line + balanced load

Phase powers are equal: $S_A = S_B = S_C$

Total 3Φ power = 3 times the per-phase power; e.g., $S_{3\Phi} = 3 * S_A$

Y-Connection assuming abc-sequence:

$\vec{V}_{ab} = (\sqrt{3}\angle 30^\circ)\vec{V}_{an}$ or $V_L = \sqrt{3}V_\phi$ [magnitude only]   Same for load end

Line current = phase current: $\vec{I}_{aA} = \vec{I}_{AN}$ or $I_L = I_\phi$ [magnitude only]

Δ-Connection assuming abc-sequence:

Δ-to-Y Transformation: $Z_Y = \frac{1}{3} Z_\Delta$

Line voltage = phase voltage = $\vec{V}_{AB}$

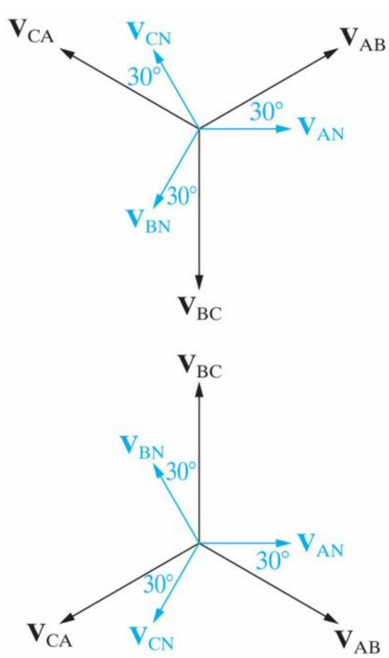
$\vec{I}_{aA} = (\sqrt{3}\angle -30^\circ)\vec{I}_{AB}$ or $I_L = \sqrt{3}I_\phi$ [magnitude only]

Three-phase complex power: $S_{3\Phi} = \sqrt{3}V_L I_L \angle\theta = P_{3\Phi} + jQ_{3\Phi}$

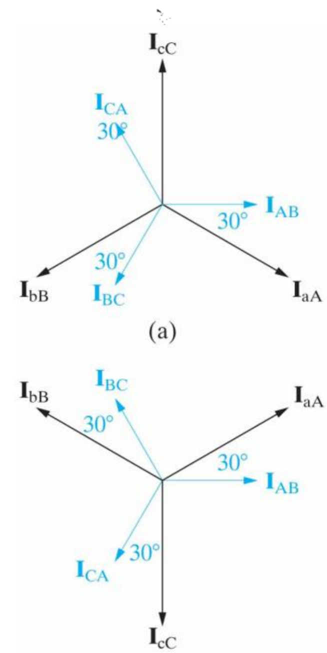
V_L = line voltage magnitude

I_L = line current magnitude

$\theta = \angle S = \angle Z = \angle\vec{V} - \angle\vec{I}$ = power factor angle



Phasor diagrams showing the relationship between line-to-line and line-to-neutral voltages in a balanced system. (a) The abc sequence. (b) The acb sequence



(b)

Phasor diagrams showing the relationship between line currents and phase currents in a Δ-connected load. (a) The positive sequence. (b) The negative sequence.