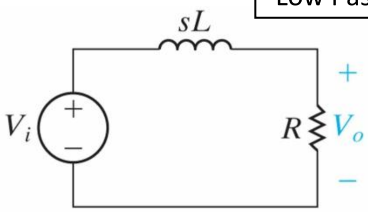
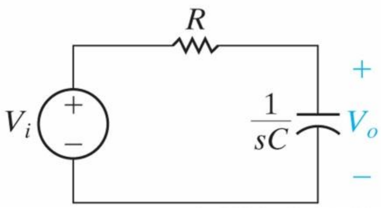
Low Pass



$$H(s) = \frac{R/L}{s + R/L}$$

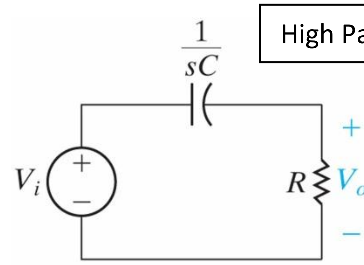
$$\omega_c = R/L$$



$$H(s) = \frac{1/RC}{s + 1/RC}$$

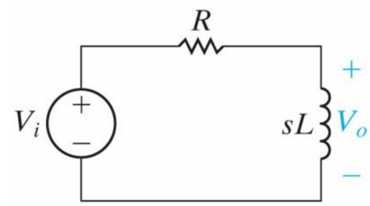
$$\omega_c = 1/RC$$

High Pass



$$H(s) = \frac{s}{s + 1/RC}$$

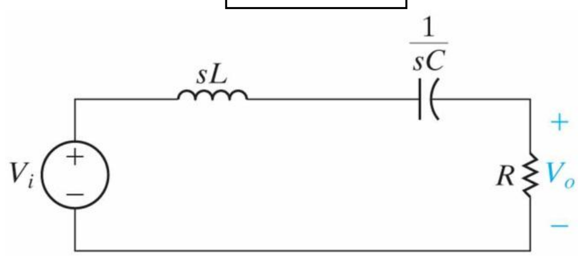
$$\omega_c = 1/RC$$



$$H(s) = \frac{s}{s + R/L}$$

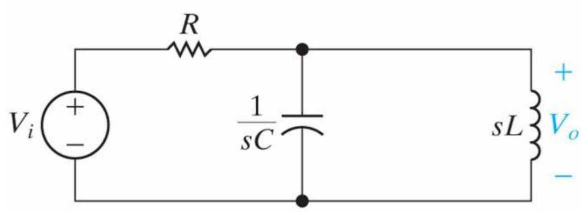
$$\omega_c = R/L$$

Band Pass



$$H(s) = \frac{(R/L)s}{s^2 + (R/L)s + 1/LC}$$

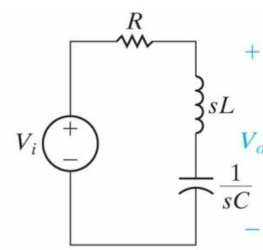
$$\omega_o = \sqrt{1/LC} \qquad \beta = R/L$$



$$H(s) = \frac{s/RC}{s^2 + s/RC + 1/LC}$$

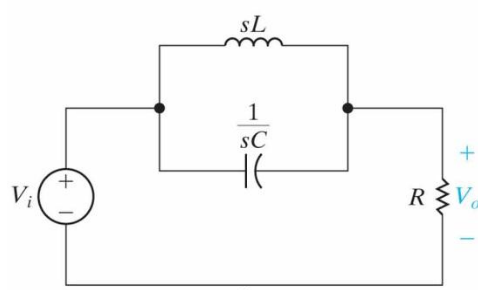
$$\omega_o = \sqrt{1/LC} \qquad \beta = 1/RC$$

Band Reject



$$H(s) = \frac{s^2 + 1/LC}{s^2 + (R/L)s + 1/LC}$$

$$\omega_o = \sqrt{1/LC} \qquad \beta = R/L$$



$$H(s) = \frac{s^2 + 1/LC}{s^2 + s/RC + 1/LC}$$

$$\omega_o = \sqrt{1/LC} \qquad \beta = 1/RC$$

Quality Factor $Q = \frac{\omega_o}{\beta}$ **Center frequency** $\omega_o = \sqrt{\omega_{c1}\ \omega_{c2}}$ **Bandwidth** $\beta = \omega_{c2} - \omega_{c1}$

Cut-off frequencies

$$\omega_{c1} = -\frac{\beta}{2} + \sqrt{\left(\frac{\beta}{2}\right)^2 + \omega_o^2} \qquad \omega_{c2} = \frac{\beta}{2} + \sqrt{\left(\frac{\beta}{2}\right)^2 + \omega_o^2}$$

Scaling Factors

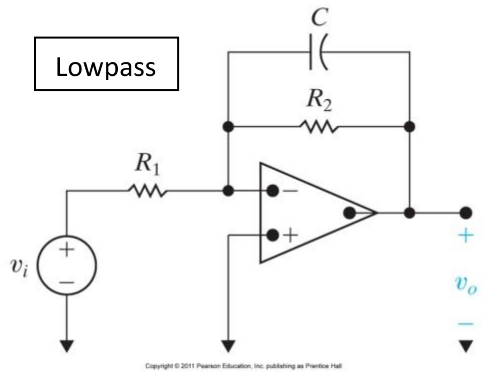
$$R' = k_m R; \quad L' = \frac{k_m}{k_f}L; \quad C' = \frac{C}{k_m k_f}; \quad k_f = \frac{\omega_c'}{\omega_c}$$

Converting to decibel

$$A_{dB} = 20\log_{10}|H(j\omega)|$$

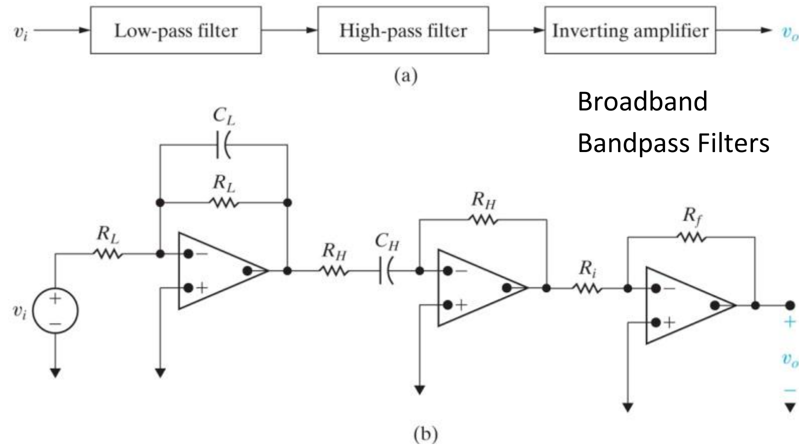
ECEN 2633 Test #3 Handout

Lowpass



$$H(s) = -\frac{\frac{R_2}{R_1}\left(\frac{1}{R_2 C}\right)}{s + \frac{1}{R_2 C}} = -K\frac{\omega_c}{s+\omega_c}$$

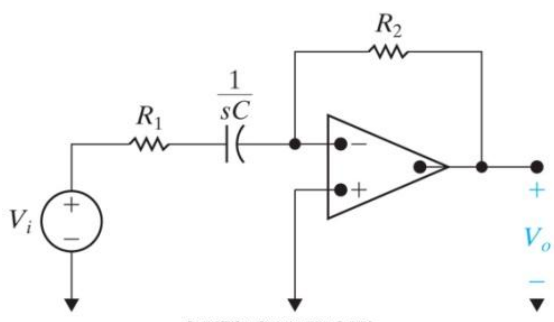
Broadband Bandpass Filters



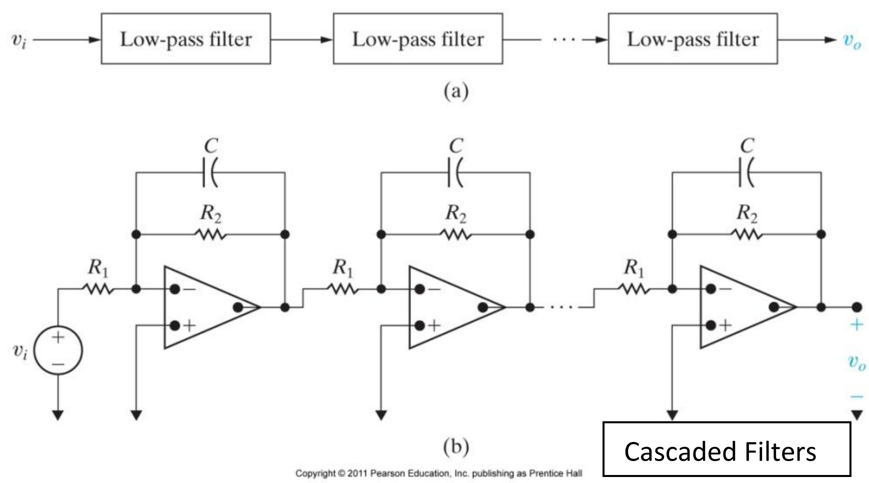
$$\omega_{c2} = {}^{1}\!/_{R_L C_L};\quad \omega_{c1} = {}^{1}\!/_{R_H C_H};\quad K = \frac{R_f}{R_i}(s);$$

$$H(s) = \frac{-K\omega_{c2}s}{s^2 + \omega_{c2}s + \omega_{c1}\omega_{c2}}$$

Highpass

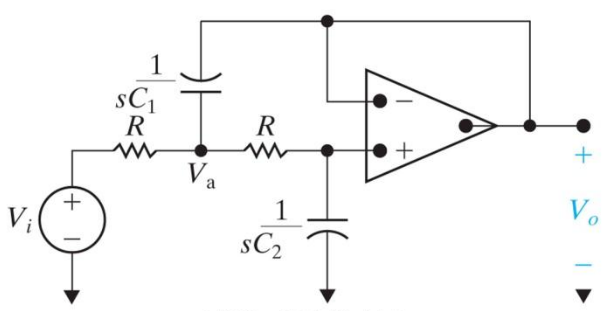


$$H(s) = -\frac{R_2}{R_1 + \frac{1}{sC}} = -K\frac{s}{s+\omega_c}$$



Cascaded Filters

$$\omega_{cn} = \sqrt{\sqrt[n]{2} - 1};\qquad k_f = \frac{\omega_c}{\omega_{cn}};\qquad |H(j\omega_{cn})| = \frac{1}{\left(\sqrt{\left({}^{\omega}\!/_{\omega_{cn}}\right)^2 + 1}\right)^2}$$



Butterworth Low-pass

$$\frac{\sqrt{10^{-0.1A_s} - 1}}{\sqrt{10^{-0.1A_p} - 1}} = \frac{\sigma_s}{\sigma_p}$$

$$n = \frac{\log_{10}\left(\frac{\sigma_s}{\sigma_p}\right)}{\log_{10}\left(\frac{\omega_s}{\omega_p}\right)} = \frac{-0.05A_s}{\log_{10}\left(\frac{\omega_s}{\omega_p}\right)}$$

$$H(s) = \frac{1}{s^2 + b_1 s + 1};$$

$$b_1 = \frac{2}{C_1}\ and\ 1 = \frac{1}{C_1 C_2}$$

$$|H(j\omega)| = \frac{1}{\sqrt{1 + \left({}^{\omega}\!/_{\omega_c}\right)^{2n}}}$$

TABLE 15.1 Normalized (so that $\omega_c = 1$ rad/s) Butterworth Polynomials up to the Eighth Order

n	nth-Order Butterworth Polynomial
1	$(s + 1)$
2	$(s^2 + \sqrt{2}s + 1)$
3	$(s + 1)(s^2 + s + 1)$
4	$(s^2 + 0.765s + 1)(s^2 + 1.848s + 1)$
5	$(s + 1)(s^2 + 0.618s + 1)(s^2 + 1.618s + 1)$
6	$(s^2 + 0.518s + 1)(s^2 + \sqrt{2} + 1)(s^2 + 1.932s + 1)$
7	$(s + 1)(s^2 + 0.445s + 1)(s^2 + 1.247s + 1)(s^2 + 1.802s + 1)$
8	$(s^2 + 0.390s + 1)(s^2 + 1.111s + 1)(s^2 + 1.6663s + 1)(s^2 + 1.962s + 1)$