Chapter 10: Sinusoidal Steady-State Power Calculations

10.1 Instantaneous Power

**Instantaneous Power**: product of the instantaneous terminal voltage and current; (positive when current is from positive to negative); the frequency of the power is twice the frequency of the voltage or current.

\[ p = vi \text{ where } v = V_m \cos(\omega t + \theta_v) \text{ and } i = I_m \cos(\omega t + \theta_i) \]

Utilizing trig. identities

\[ \cos \alpha \cos \beta = \frac{1}{2} \cos(\alpha - \beta) + \frac{1}{2} \cos(\alpha + \beta) \] and \[ \cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta \]

\[ p = \frac{V_m I_m}{2} \cos(\theta_v - \theta_i) + \frac{V_m I_m}{2} \cos(\theta_v + \theta_i) \cos 2\omega t - \frac{V_m I_m}{2} \sin(\theta_v - \theta_i) \sin 2\omega t \]

10.2 Average and Reactive Power

**Average Power**: the average value of the instantaneous power over one period; power converted from from electrical to nonelectrical form and vice versa: often referred to as the real power.

**Reactive Power**: electrical power exchanged between the magnetic field of an inductor and the source that drives it or between the electric field of a capacitor and the source that drives it. (Reactive power is never converted to nonelectric power)

Rewriting the three terms of the instantaneous power

\[ p = P + P \cos 2\omega t - Q \sin 2\omega t \]

Where

\[ P = \frac{V_m I_m}{2} \cos(\theta_v - \theta_i) \text{ = the average (real) power} \]
\[ Q = \frac{V_m I_m}{2} \sin(\theta_v - \theta_i) \text{ the reactive power} \]
Chapter 10: Sinusoidal Steady-State Power Calculations

Purely resistive circuits: \( \theta_v = \theta_i \)

\[ p = P + P \cos 2\omega t \]

Graphical representation assuming \( \omega = 377 \text{ rad/s} \)

Purely inductive circuits: \( \theta_i = \theta_v - 90^\circ \)

(Current lags voltage by 90°)

\[ p = -Q \sin 2\omega t \]

Reactive power is given in units of \( \text{vars} \) (volt-amp reactive)

Graphical representation assuming \( \omega = 377 \text{ rad/s} \)
(Note: average power is zero thus no energy transformation)

Purely capacitive circuits: \( \theta_v - \theta_i = -90^\circ \)

(Current leads voltage by 90°)

\[ p = -Q \sin 2\omega t \]

Graphical representation assuming \( \omega = 377 \text{ rad/s} \)

Note: Inductors demand (absorb) vars; capacitor furnish (deliver) vars

Power factor (pf): cosine of the phase angle between the voltage and the current

\[ \text{pf} = \cos(\theta_v - \theta_i) \]

Lagging power factor: implies that the current lags the voltage – hence inductive load

Leading power factor: implies that the current leads the voltage – hence capacitive load

Reactive factor (rf): sine of the phase angle between the voltage and the current

\[ \text{rf} = \sin(\theta_v - \theta_i) \]

Review Example 10.1 & 10.2 and Assessment Problem 10.1 & 10.2

10.3 The RMS Value of Power Calculations

From chapter 9: RMS is square root of the mean of the square of the function
Chapter 10: Sinusoidal Steady-State Power Calculations

\[ V_{rms} = \frac{1}{T} \int_{t_0}^{t_0+T} V_m^2 \cos^2(\omega t + \phi) \, dt = \frac{V_m}{\sqrt{2}} \]

Average power can be written as

\[ P = \frac{1}{T} \int_{t_0}^{t_0+T} \frac{V_m^2}{R} \cos^2(\omega t + \phi) \, dt = \frac{V_{rms}^2}{R} = I_{rms}^2 R \]

**Effective value:** another name given to the RMS value

A DC source delivers the same energy over a given time as a sinusoidal source with the same rms value, assuming same load resistance.

Rewriting

\[ P = \frac{V_m I_m}{2} \cos(\theta_v - \theta_i) = \frac{V_m}{\sqrt{2} \sqrt{2}} \cos(\theta_v - \theta_i) = V_{eff} I_{eff} \cos(\theta_v - \theta_i) \]

Therefore

\[ Q = V_{eff} I_{eff} \sin(\theta_v - \theta_i) \]

Note: Effective values of sinusoids are widely used in rating equipment (i.e. 240/120V services)

Ex. The current used by a 100W light bulb in a 120V service (residential)

\[ R = \frac{120^2}{100} = 144; \quad i = \frac{120}{144} = 0.833A \text{ rms} \text{ or } 0.833\sqrt{2} = 1.18A \text{ peak} \]

**Review Example 10.3 and Assessment Problem 10.3**

10.4 Complex Power

Complex power (S): sum of the real and reactive powers given in volt-amps (VA)

\[ S = P + jQ \]

**Apparent Power:** magnitude of the complex power

\[ |S| = \sqrt{P^2 + Q^2} \]

**TABLE 10.2 Three Power Quantities and Their Units**

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Complex power</td>
<td>volt-amps</td>
</tr>
<tr>
<td>Average power</td>
<td>watts</td>
</tr>
<tr>
<td>Reactive power</td>
<td>var</td>
</tr>
</tbody>
</table>

**Review Example 10.4**
Chapter 10: Sinusoidal Steady-State Power Calculations

10.5 Power Calculations

Complex power
\[ S = \frac{V_m I_m}{2} \cos(\theta_v - \theta_i) + j \frac{V_m I_m}{2} \sin(\theta_v - \theta_i) = \frac{V_m I_m}{2} [\cos(\theta_v - \theta_i) + j\sin(\theta_v - \theta_i)] \]
\[ = \frac{V_m I_m}{2} e^{j(\theta_v - \theta_i)} = \frac{1}{2} V_m I_m \angle (\theta_v - \theta_i) \]

RMS
\[ S = V_{eff} I_{eff} \angle (\theta_v - \theta_i) = \overline{V_{eff} I_{eff}} \]

Phasor
\[ S = \frac{1}{2} \overline{V I^*} \]

Given
\[ \overline{V_{eff}} = Z I_{eff} \]

\[ S = Z I_{eff} I_{eff}^* = |I_{eff}|^2 Z = P + jQ \]

Therefore
\[ P = |I_{eff}|^2 R = \frac{1}{2} I_m^2 R = \frac{|V_{eff}|^2}{R} \]
\[ Q = |I_{eff}|^2 X = \frac{1}{2} I_m^2 X = \frac{|V_{eff}|^2}{X} \]

Review Example 10.5 – 10.7 and Assessment Problem 10.4 – 10.6

10.6 Maximum Power Transfer

\[ Z_L = Z_{Th}^* \]

Maximum power
\[ P_{max} = \left( \frac{|V_{Th}|}{2R_L} \right)^2 R_L = \frac{1}{4} \frac{|V_{Th}|^2}{R_L} \]

If voltage is not in RMS values then
\[ P_{max} = \frac{1}{8} \frac{V_m^2}{R_L} \]

Review Example 10.8 – 10.10 and Assessment Problem 10.7